KRYLOV SUBSPACE METHODS FOR AN INITIAL VALUE PROBLEM ARISING IN TEM

Martin Afanasjew
Institut für Numerische Mathematik und Optimierung,
TU Bergakademie Freiberg, 09596 Freiberg, Germany,
e-mail: martin.afanasjew@math.tu-freiberg.de

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Abstract

A popular geophysical exploration technique, known as Transient Electromagnetics (TEM) cf. [1], is based on observing the decay of a low-frequency electromagnetic field in the subsurface in order to infer geological properties of interest. The governing equations for this phenomenon are the time-dependent Maxwell's equations in the diffusive limit of vanishing displacement currents. The typical configuration of a current shutoff leads to a linear constant-coefficient initial value problem for the electric field

$$E_t = -\sigma\Big(\nabla \times (\mu^{-1} \nabla \times E)\Big), \qquad E(t_0) = E_0,$$

which, after discretization by e.g. the Yee finite difference scheme or Nédélec finite elements, becomes the ODE system

$$u'(t) = -Au, \quad u(t_0) = u_0, \tag{1}$$

in which the matrix A denotes the discrete approximation of the differential operator $\sigma(\nabla \times (\mu^{-1}\nabla \times \cdot))$. The ODE system (**??**) is typically solved using explicit time-stepping algorithms such as the Du Fort-Frankel scheme (see, e.g., [1]). Other possible methods include ODE solvers designed especially for parabolic initial value problems such as Runge-Kutta-Chebyshev methods.

In this talk we present some comparisons of such time-stepping techniques with Krylov subspace approximations of the matrix exponential $u(t) = \exp(-A(t-t_0))u_0$, including in particular restarted variants such as proposed in [2].

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References

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