

A SCALABLE MULTI-LEVEL PRECONDITIONER FOR MATRIX-FREE μ -FINITE ELEMENT ANALYSIS OF HUMAN BONE STRUCTURES

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Abstract

The recent advances in microarchitectural bone imaging are disclosing the possibility to assess both the apparent density and the trabecular microstructure of intact bones in a single measurement. Coupling these imaging possibilities with microstructural finite element (μ FE) analysis offers a powerful tool to improve bone stiffness and strength assessment for individual fracture risk prediction.

Many elements are needed to accurately represent the intricate microarchitectural structure of bone; hence, the resulting μ FE models possess a very large number of degrees of freedom. In order to be solved quickly and reliably on state-of-the-art parallel computers, the μ FE analyses require advanced solution techniques. In this paper, we investigate the solution of the resulting systems of linear equations by the conjugate gradient algorithm, preconditioned by aggregation-based multigrid methods. We introduce a variant of the preconditioner that does not need assembling the system matrix but uses element-by-element techniques to build the multilevel hierarchy. The preconditioner exploits the voxel approach that is common in bone structure analysis, it has modest memory requirements, while being at the same time robust and scalable. Using the proposed methods, we have solved in less than 10 minutes a model of trabecular bone composed of 247'734'272 elements, leading to a matrix with 1'178'736'360 rows, using only 1024 CRAY XT3 processors.

References

- [1] P. ARBENZ, G. H. VAN LENTHE, U. MENNEL, R. MÜLLER, AND M. SALA, *A scalable multi-level preconditioner for matrix-free μ -finite element analysis of human bone*

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