Rate of convergence of a new block matrix factorization variable preconditioning method

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Classical block-matrix approximate factorization methods for matrices in a two-by-two block form, corresponding to a partitioning of a finite element mesh or of a matrix graph in some fine-coarse set of points, involve at each iteration step two solutions of systems with the approximation of the pivot block, in addition to the solution of the system approximating the Schur complement with respect to the pivot block.

A new method is presented where, in addition to the Schur complement preconditioner, only one solution of the approximate pivot block (or a matrix-vector multiplication with its approximate inverse) is required. This method assumes, however, an initial transformation with a fixed approximation of the off-diagonal block matrix.

Additionally, the method has the advantage that both the outer and inner iterations can be performed using a conjugate gradient method. The use of such inner iterations leads to a variable preconditioner and therefore generalized conjugate gradient methods must be used.

The method is not only applicable for symmetric positive definite systems but also for nonsymmetric and indefinite problems of saddle point form. It is shown that the preconditioned matrix can be written in a form which readily enables estimates of the rate of convergence of the method.

Some numerical tests illustrate the method and show its robustness.