

# ON MULTIGRID METHODS FOR THE CAHN–HILLIARD EQUATION WITH OBSTACLE POTENTIAL

Ľubomír Bañas, Robert Nürnberg

*Department of Mathematics, Imperial College London, South Kensington  
Campus, SW72AZ London, United Kingdom,*

*e-mail: l.banas@imperial.ac.uk, robert.nurnberg@imperial.ac.uk*

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## Abstract

Numerical simulations of the Cahn–Hilliard equation with obstacle potential require the solution of a saddle-point system with inequality constraint. Based on a preconditioned Uzawa-Multigrid algorithm that has been proposed in [1], the authors recently developed an efficient solution method for a finite element approximation of a degenerate Cahn–Hilliard equation in 3D, see [2]. The Uzawa-Multigrid algorithm can be viewed as an active set strategy, that requires the solution of an unconstrained saddle-point problem. The unconstrained problem can be effectively solved by standard multigrid methods for linear saddle-point problems, however an outer iteration is needed to obtain an approximation of the active set. Here we present a new full multigrid method that is applied directly to the original problem with the inequality constraint, i.e. no outer iterations are needed. Finally, we present some 3D experiments where we illustrate the performance of both multigrid methods.

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## References

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