MODEL REDUCTION BY A CROSS-GRAMIAN APPROACH FOR DATA-SPARSE SYSTEMS

Ulrike Baur, Peter Benner
Fakultät für Mathematik, Technische Universität Chemnitz, Germany

Keywords: model reduction, hierarchical matrices, Sylvester equations, Cross-Gramian

Abstract

We consider linear time-invariant (LTI) systems of the following form

$$\Sigma : \begin{cases} \dot{x}(t) & = & Ax(t) + Bu(t), \qquad t > 0, \qquad x(0) = x^0, \\ y(t) & = & Cx(t) + Du(t), \qquad t \geq 0, \end{cases}$$

with stable state matrix $A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times m}$, $C \in \mathbb{R}^{p \times n}$, $D \in \mathbb{R}^{p \times m}$, arising, e.g., from the discretization and linearization of parabolic PDEs. Typically, in practical applications, we have a large state-space dimension $n = \mathcal{O}(10^5)$ and a small input and output space, $n \gg m$, p. We further assume that the system is square, i.e., $p = m$. We show how to compute an approximate reduced-order system $\hat{\Sigma}$ of order $r \ll n$ with a balancing-related model reduction method. The method is based on the computation of the cross-Gramian X, which is the solution of the Sylvester equation

$$AX + XA + BC = 0.$$

As standard algorithms for the solution of Sylvester equations are of limited use for large-scale systems, we investigate approaches based on the matrix sign function method [2]. To make this iterative method applicable in the large-scale setting, we propose a modified iteration scheme for computing low-rank factors of the solution X and we incorporate structural information from the underlying PDE model into the approach. By using data-sparse matrix approximations, hierarchical matrix formats, and the corresponding formatted arithmetic we obtain an efficient solver having linear-polylogarithmic complexity [1]. We show that the reduced-order model can then be computed from the low-rank factors directly.

Note this continues the talk submitted by P. Benner.

References

[1] U. Baur, *Low rank solution of data-sparse Sylvester equations*, Preprint #266, MATHEON, Berlin, FRG, Oct. 2005. To appear in Numerical Linear Algebra with Applications.

[2] P. Benner, *Factorized solution of Sylvester equations with applications in control*, In Proc. Intl. Symp. Math. Theory Networks and Syst 2004.