NUMERICAL RANGE ERROR ESTIMATES FOR EVALUATING FUNCTIONS OF MATRICES VIA THE ARNOLDI METHOD

Bernhard Beckermann
Laboratoire Paul Painlevé UMR 8524
UFR de Mathématiques, Université de Lille 1, F-59655 Villeneuve d'Ascq, France,
e-mail: bbecker@math.univ-lille1.fr

Lothar Reichel
Department of Mathematical Sciences,
Kent State University, Kent, OH 44242,
e-mail: reichel@math.kent.edu

Keywords: Arnoldi method, numerical range, Faber polynomials

Abstract

In this talk we propose explicit a priori error bounds for approaching $f(A)b$ by help of the Arnoldi method. Here A is a large real not necessarily symmetric matrix, and f some function analytic on the field of values or numerical range $W(A) = \{y^*Ay : \|y\| = 1\}$. An essential tool in our work is the inequality $\|F_n(A)\| \leq 2$ derived in [1] where F_n is the nth Faber polynomial corresponding to $W(A)$, and $\|\cdot\|$ denotes euclidean vector norms and the induced spectral matrix norm. We show in a first step how to improve bounds given by Knizhnerman [3] and by Hochbruck and Lubich [2]. Subsequently we give some simple bounds in terms of the numerical range for the exponential function as well as for Stieltjes functions like the pth power of A.

References

[1] B. Beckermann, *Image numérique, GMRES et polynômes de Faber*, C. R. Acad. Sci. Paris, Ser. I, 340 (2005), pp. 855–860.

[2] M. Hochbruck, Ch. Lubich, *On Krylov subspace approximations to the matrix exponential operator*, SIAM J. Numer. Anal., 34 (1997), pp. 1911–1925.

[3] L. A. Knizhnerman, *Calculation of functions of unsymmetric matrices using Arnoldi's method*, Comput. Math. Math. Phys., 31, 1 (1991) pp. 1–9 (in English); translation from Zh. Vychisl. Mat. Mat. Fiz., 31, 1 (1991), pp. 3–16 (in russian).