ORTHOGONALIZATION VIA DEFLATION: A MINIMUM NORM APPROACH FOR LOW - RANK APPROXIMATIONS OF A MATRIX

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Abstract

In this talk we introduce a new orthogonalization method. Given a real $m \times n$ matrix A, the new method constructs an SVD-type decomposition of the form $A = U\Sigma V^T$. The columns of U and V are orthogonal, or nearly orthogonal, while Σ is a diagonal matrix whose diagonal entries approximate the singular values of A. The method has three versions. A "left-side" orthogonalization scheme in which the columns of U constitute an orthonormal basis of Range(A). A "right-side" orthogonalization scheme in which the columns of R constitute an orthonormal basis of Range (A^T) . In the third version both U and V have orthonormal columns, but the decomposition is not exact.

Starting from $A_1 = A$ the deflation process generates a sequence of matrices A_1, A_2, A_3, \ldots , by the rule

$$A_{k+1} = A_k - \sigma_k \mathbf{u}_k (\mathbf{v}_k)^T, \quad k = 1, 2, 3, \dots$$

where $\{\sigma_k, \mathbf{u}_k, \mathbf{v}_k\}$ denotes a computed estimate of a singular triplet of A_k . The estimated singular vectors are obtained by a few "rectangular iterations" for solving the minimum norm problem

minimize
$$F(\mathbf{u}, \mathbf{v}) = ||A_k - \mathbf{u}\mathbf{v}^T||_F^2$$
.

The resulting orthogonal decomposition may substitute the SVD in many applications. The advantage of the new method lies in problems with missing data. That is, when some entries of A are unknown. Standard SVD algorithms are unable to handle such matrices. Yet the minimum norm approach overcomes this difficulty in an elegant way: The objective function is redefined as

$$F(\mathbf{u}, \mathbf{v}) = \Sigma (a_{ij} - u_i * v_j)^2,$$

where the sum is restricted to known entries of A. This modification enables us to construct a low-rank approximation of A, or a "pseudo SVD" of A, in spite of the missing data. Once a pseudo SVD of A is constructed, it can be used for estimating the missing data. Numerical experiments illustrate the usefulness of the proposed methods.