POTENTIAL THEORY AND PRACTICAL ASPECTS OF THE SOLUTION OF LYAPUNOV EQUATIONS

Mark Embree
Computational and Applied Mathematics, Rice University
6100 Main Street – MS 134, Houston, Texas, 77005–1892, USA
e-mail: `embree@rice.edu`

John Sabino
The Boeing Company, Seattle, Washington
P.O. Box 3707, Seattle, Washington, 98124, USA

Keywords: ADI iteration, condenser capacity, Sylvester equation

Abstract

The Lyapunov equation $AX+XA^* = -BB^*$ poses considerable computational challenges when the dimension of A is large, for the solution X is typically dense regardless of the sparsity of A. However, in many applications the right-hand side has low rank, and under broad hypotheses the solution X inherits this trait. Effective algorithms, such as the modified Smith method, efficiently approximate X in a low-rank factored form.

Numerous issues must be addressed to translate this observation into an algorithm viable for the kind of large-scale problems that arise, for example, in balanced truncation model reduction. In this talk, we shall describe estimates on the decay of the singular values of X derived from potential theory (work in collaboration with Christopher Beattie). Such analysis suggests both the rank of X and asymptotically optimal shifts for use in the modified Smith method. Practical considerations naturally restrict the number of shifts, and we shall demonstrate how one can often achieve more effective convergence by repeatedly applying a small set of shifts determined via an optimization scheme. Several other important considerations, ranging from the order in which shifts are applied and the use of inexact inner iterations, to techniques for efficiently computing the residual norm [1], shall also be discussed.

Acknowledgement: This research was supported in part by US National Science Foundation Grant DMS-CAREER-0449973.

References

[1] John Sabino, *Solution of large-scale Lyapunov equations via the block modified Smith method*, Ph.D. Thesis, Rice University, 2006.