MULTILEVEL KRYLOV METHOD FOR THE HELMHOLTZ EQUATION

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Abstract

In the first part of the talks on multilevel Krylov methods, Reinhard Nabben discussed the underlying concept of the method and showed by some numerical examples the effectiveness of the method. In this talk, we extend the application of the multilevel Krylov method to the indefinite, high wavenumber Helmholtz equation.

In this case, we consider the preconditioned Helmholtz system, where the preconditioner is the discrete formulation of the shifted Laplace preconditioner [1] and is solved by one multigrid iteration. With this preconditioning, the eigenvalues of the system are clustered around zero and one. To speed up the convergence, multilevel Krylov method is applied to the preconditioned system, based on shifting of small eigenvalues to one. To construct the coarse grid problem associated with the projected small eigenvalues, an approximation based on a product of low dimension matrices is introduced. This approximation will require a multigrid iteration of reduced level in the preconditioning step at every level in the projection step.

Numerical examples show that the convergence of Krylov iteration applied to 2D high wavenumber Helmholtz problems can be made independent of both grid size and wavenumber.

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References

[1] Y. A. Erlangga, C.W. Oosterlee, and C. Vuik, *A novel multigrid-based preconditioner for the heterogeneous Helmholtz equation*, SIAM J. Sci. Comput., 27 (2006), pp. 1471–1492.