KARHUNEN-LOÈVE APPROXIMATION OF RANDOM FIELDS USING HIERARCHICAL MATRIX TECHNIQUES

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Abstract

In stochastic finite element computations for modelling uncertainty a popular approach for separating the deterministic and stochastic dependencies of a random field is to compute the first few terms of its Karhunen-Loève (KL) expansion. This entails approximating the dominant eigenpairs of its covariance operator, leading to a large dense eigenvalue problem, in particular since the operator typically acts on functions defined on a domain, not only its boundary.

In this talk we emphasize the relation of the KL expansion to the singular value decomposition and present our experiences in computing approximate covariance eigenpairs based on Galerkin discretization and Krylov subspace projection. The latter requires efficient matrix-vector multiplication routines. Recent work has explored fast multipole techniques [2]. Here we employ the hierarchical matrix approximation technique [1].

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References

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