ON A QUADRATIC EIGENVALUE PROBLEM ARISING IN THE ANALYSIS OF DELAY EQUATIONS

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Abstract

The analysis of retarded linear m-delay time delay systems

$$\begin{array}{ll} \dot{x}(t) = \sum_{k=0}^{m} A_k x(t-h_k), & t > 0 \\ x(t) = \phi(t), & t \in [-h_m, 0] \end{array}$$

with $h_0 = 0 < h_1 < \ldots < h_m, x : [-h_m, \infty) \to \mathbb{R}^n, A_k \in \mathbb{R}^{n \times n}$, leads to a quadratic eigenvalue problem $Q(\lambda)u = 0$ where

$$Q(\lambda) = \lambda^2 E + \lambda F + G$$

with $E = A_m \otimes I, G = I \otimes A_m$, and $F = \sum_{k=0}^{m-1} I \otimes A_k e^{-i\phi_k} + A_k \otimes I e^{i\phi_k}, \phi_k = \omega h_k$ where $\otimes$ denotes the Kronecker product.

As there exists a permutation matrix P such that $P^T(A \otimes B)P = B \otimes A$ for all real $n \times n$ matrices A, B, the quadratic matrix polynomial Q satisfies

$$P^T \mathrm{rev}(\overline{Q}(\lambda))P = Q(\lambda), \tag{1}$$

where $\overline{Q}(\lambda) = \lambda^2 \overline{E} + \lambda \overline{F} + \overline{G}$ and $\mathrm{rev}(Q(\lambda)) = \lambda^2 Q(\frac{1}{\lambda})$. Matrix polynomials which satisfy (**??**) remind of the different palindromic polynomial definition given in [1], e.g., a palindromic polynomial is given by $\mathrm{rev}(Q(\lambda)) = Q(\lambda)$, while a $\star$-palindromic polynomial satisfies $\mathrm{rev}(Q^\star(\lambda)) = Q(\lambda)$, where $\star$ is used as an abbreviation for transpose T in the real case and either T or conjugate transpose $*$ in the complex case.

Following the derivations in [1], we will discuss the spectral symmetry of matrix polynomials (**??**) as well the structured linearizations where we continue the practise stemming from Lancaster of developing theory for polynomials of degree k where possible in order to gain the most insight and understanding.

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References

[1] D.S. Mackey, N. Mackey, C. Mehl, and V. Mehrmann, *Structured polynomial eigenvalue problems: Good vibrations from good linearizations*, SIAM J. Matrix Anal. Appl., 28 (2006), pp. 1029–1051.