UPPER AND LOWER BOUNDS ON THE NORMS OF FUNCTIONS OF MATRICES

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Abstract

Given an n by n matrix A, we look for a set S in the complex plane and positive constants m and M such that for all polynomials (or analytic functions) p, the inequalities

$$m \cdot \inf\{\|f\|_{\mathcal{L}^\infty(S)} : \ f(A) = p(A)\} \leq \|p(A)\| \leq M \cdot \inf\{\|f\|_{\mathcal{L}^\infty(S)} : \ f(A) = p(A)\}$$

hold. We show that for 2 by 2 matrices, if S is the field of values, then one can take $m = 1$ and $M = 2$. We show that for a perturbed Jordan block – a matrix A that is an n by n Jordan block with eigenvalue 0 except that its $(n, 1)$-entry is $\nu \in (0, 1)$ – if S is the unit disk, then $m = M = 1$. More generally, we show that if A is a companion matrix whose eigenvalues lie in the open unit disk $\mathcal{D}$, then $m = 1$ if $S = \mathcal{D}$. We argue, however, that if S is simply connected, then in most cases, in order to have a moderate size constant m, there must be a one-to-one analytic mapping from S onto $\mathcal{D}$ that maps almost all of the eigenvalues to points near the boundary of $\mathcal{D}$.