STABILIZATION OF RESTARTED KRYLOV METHODS FOR MATRIX FUNCTIONS

Stefan Güttel
Institut für Numerische Mathematik und Optimierung,
TU Bergakademie Freiberg, 09596 Freiberg, Germany,
e-mail: stefan.guettel@math.tu-freiberg.de

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Abstract

In [1] Eiermann and Ernst proposed a restarting scheme for Krylov subspace approximations to $f(A)b$ where f is a suitable function, A a square matrix and b a vector. The approximates are of the form

$$f(A)b \approx \beta V_m f(H_m) e_1,$$

where

$$AV_m = V_m H_m + \eta_{m_1,m} v_{m+1} e_1^\top$$

is an Arnoldi-like decomposition and $\beta v_1 = b$. The most straightforward implementation is based on the evaluation of an expression for the error which involves repeated divided differences of f where the nodes are the Ritz values associated with the successive Krylov subspaces generated in the course of the restart algorithm. Since the direct evaluation of this error formula is in general unstable, a somewhat more expensive variant was proposed in [1] which required applying the function f to an aggregate Hessenberg matrix and which was found to be stable.

In this talk we present an alternative stable variant based on the Schur-Parlett approach for evaluating matrix functions by solving Lyapunov equations. This variant is stabilized by repeated swapping of diagonal blocks inside the Schur form (cf. [2]).

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References

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[2] Z. Bai and J. W. Demmel, *On swapping diagonal blocks in real Schur form*, Linear Algebra Appl. 186 (1993), pp. 73–95.