# IMAGE DEBLURRING IN THE LIGHT OF DCT

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**Abstract**

In the setting of matrix computations, the model for the blurring of the image is $A\,x = b$, where the vectors $x$ and $b$ represent the exact and blurred images, and the matrix $A$ represents the blurring process. Since image deblurring is a discrete ill-posed problem, it is necessary to use regularization in order to compute stable solutions [1]. Moreover, it is often advantageous to impose boundary conditions on the reconstruction, which is achieved by a simple modification of the coefficient matrix [2], [3].

This paper focuses on *regularizing iterations* where we apply a Krylov subspace method directly to the problem $Ax = b$. The regularization comes from the projection of the solution on the Krylov subspace associated with the method, and the number of iterations plays the role of the regularization parameter.

We use the two-dimensional discrete cosine transform (DCT) to perform a spectral analysis of the solutions to the image deblurring problem, computed by means of regularizing iterations, and we focus on CGLS/LSQR and GMRES and their variants MINRES, RRGMRES and MR-II. To the best of our knowledge, a thorough study of the spectral and visual quality of the reconstructions computed by these methods has not been carried out.

## References

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