GMRES METHODS FOR LEAST SQUARES PROBLEMS

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Abstract

The standard iterative method for solving large sparse least squares problems $\min_{\boldsymbol{x}\in\mathbf{R}^n} \|\boldsymbol{b} - A\boldsymbol{x}\|_2$, $A \in \mathbf{R}^{m\times n}$ is the CGLS method, or its stabilized version LSQR, which applies the (preconditioned) conjugate gradient method to the normal equation $A^{\mathrm{T}}A\boldsymbol{x} = A^{\mathrm{T}}\boldsymbol{b}$.

In this talk, we will consider alternative methods [1] using a matrix $B \in \mathbf{R}^{n\times m}$ and applying the Generalized Minimal Residual (GMRES) method to $\min_{\boldsymbol{z}\in\mathbf{R}^m} \|\boldsymbol{b} - AB\boldsymbol{z}\|_2$ or $\min_{\boldsymbol{x}\in\mathbf{R}^n} \|B\boldsymbol{b} - BA\boldsymbol{x}\|_2$.

First, we give a sufficient condition concerning B for the GMRES methods to give a least squares solution without breakdown for arbitrary $\boldsymbol{b}$, for over-determined, under-determined and possibly rank-deficient problems. Next, we give a convergence analysis of the GMRES methods as well as the CGLS method. Then, we propose using the robust incomplete factorization (RIF) [2] for B.

Finally, we show by numerical experiments on over-determined and under-determined problems that, for large problems, the GMRES methods with RIF, give least squares solutions faster than the CGLS and LSQR methods with RIF. We will also discuss the effect of reorthogonalization on the CGLS method.

References

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