NUMERICAL ANALYSIS OF THE MATRIX LOGARITHM

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Abstract

We present the theory of the matrix logarithm and algorithms for computing it and its condition number. The matrix logarithm arises in a number of applications and we begin by outlining some of them. We classify all logarithms of a matrix, analyze when $\log(AB) = \log(A) + \log(B)$ for matrices A and B, and characterize the Fréchet derivative and the condition number. The inverse scaling and squaring method based on Padé approximation and repeated square roots is then developed for both triangular and full matrices and compared with a Schur–Parlett algorithm. Finally, numerical evaluation of the Fréchet derivative and exact computation and estimation of the condition number are treated.

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