STABILIZING ITERATIVE METHODS: LIMITS OF PERFORMANCE VIA REACHABLE SETS

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Abstract

Numerical algorithms can be interpreted as discrete-time control systems, incorporating shift parameters as control variables. We analyze the geometric structure of the reachable sets for two well-known algorithms: GMRES(m) and Rayleigh Iteration. Our results provide fundamental limitations on the possibility of feedback stabilization. Necessary as well as sufficient conditions for the existence of convergent shift strategies are derived.

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