A TECHNIQUE FOR COMPUTING MINORS OF ORTHOGONAL MATRICES

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Abstract

We introduce a technique for computing all possible principal determinants (minors) of order $(n-j)\times(n-j)$ of various orthogonal $n\times n$ matrices, which works also for orthogonal matrices in a generalized sense, e.g. for binary Hadamard matrices S ($S^TS=\frac{1}{4}(n+1)(I_n+J_n)$). The method takes advantage of the orthogonality property $A^TA=kI_n$ of an orthogonal matrix A and of the symmetry of B^TB, where B is a principal submatrix of A. The idea is facilitated by algebraic computations based on determinant formulas for matrices of the form $(k-\lambda)I_n+\lambda J_n$ and for block matrices of the form $\left[\begin{smallmatrix} B_1 & B_2 \\ B_3 & B_4 \end{smallmatrix}\right]$. The whole process can be standardized and implemented as a computer algorithm, which overcomes the difficulties occurring due to strenuous calculations done by hand. Theoretically, the algorithm works for every values n and j, and specifically for small values of j it provides general (i.e. they are valid of general n), analytical formulas. The symbolic implementation of the algorithm guarantees its precision. Finally, we justify the usefulness of such a method with an application to a problem of Numerical Linear Algebra, the growth problem. The results presented here are a generalization of an idea discussed in [2] for larger values j and also for other classes of orthogonal matrices. Extensive information on orthogonal matrices can be found in [1].

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References

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