AN ALGORITHM FOR SOLUTION OF NON-SYMMETRIC SADDLE-POINT SYSTEMS

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Abstract

The contribution deals with fast solving of non-symmetric saddle-point systems

$$\begin{pmatrix} A & B_1^\top \\ B_2 & 0 \end{pmatrix} \begin{pmatrix} u \\ \lambda \end{pmatrix} = \begin{pmatrix} f \\ g \end{pmatrix}, \tag{1}$$

where an $(n \times n)$ diagonal block A is possibly singular and $(m \times n)$ off-diagonal blocks B_1, B_2 have full row-rank and they are highly sparse. We will be interested especially in systems (**??**) with n large, m much smaller than n and with the defect l of A, $l = n - rankA$, much smaller than m.

Our algorithm is based on the Schur complement reduction. If A is singular, the reduced system has again the saddle-point structure (**??**), however its size is considerably smaller. After applying orthogonal projectors, we obtain an equation in terms of λ only that can be solved by a *projected* Krylov subspace method for non-symmetric operators. For this purpose, we derive a projected variant of the BiCGSTAB algorithm from the non-projected one, whose iterations can be accelerated by a multigrid strategy.

The presented method can be viewed as a generalization of algebraic ideas used in FETI domain decomposition methods [1], where A is symmetric positive semidefinite and $B_1 = B_2$.

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References

[1] C. Farhat, J. Mandel, F. X. Roux, *Optimal convergence properties of the FETI domain decomposition method*, Comput. Methods Appl. Mech. Engrg., 115 (1994), pp. 365–385.