COMPUTING SPARSE SOLUTIONS OF UNDERDETERMINED STRUCTURED SYSTEMS BY GREEDY METHODS

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Keywords: underdetermined systems, compressive sampling, greedy methods, sparse Fourier analysis, FFT

Abstract

Recently, the surprising fact that it is possible to recover functions having only few non-zero coefficients with respect to some basis from vastly incomplete information has gained much attention. Such functions are commonly called sparse or compressible and they naturally appear in a wide range of applications. We study *sparse trigonometric polynomials*

$$f(x) = \sum_{k \in I_N} \hat{f}_k \mathrm{e}^{-2\pi \mathrm{i} k x}, \qquad I_N := \{-\frac{N}{2}, \ldots, \frac{N}{2} - 1\},$$

with non-zero Fourier coefficients $\hat{f}_k \in \mathbb{C}$ only on a set $\Omega \subset I_N$ with size $|\Omega| \ll N$. However, a priori nothing is known about Ω apart from a maximum size. Our aim is to sample f at M randomly chosen nodes $x_j \in [-\frac{1}{2}, \frac{1}{2}]$ and try to reconstruct f from these samples.

Thus, we wish to solve the strongly underdetermined consistent linear system

$$A\hat{f} = y \qquad \left(a_{j,k} = \mathrm{e}^{-2\pi \mathrm{i} k x_j},\ y_j = f(x_j),\ j = 1, \ldots, M,\ k \in I_N\right),$$

for the vector $\hat{f} = (\hat{f}_k)_{k \in I_N}$ with the smallest number of non-zero entries.

For an appropriate number M of samples, greedy methods like *(Orthogonal) Matching Pursuit* or *Thresholding* succeed in this task with high probability. We focus on the computational complexity of the proposed methods when using the nonequispaced FFT and particular updating techniques. Illustration of our observations is given by numerical experiments.