# STABILITY OF KRYLOV SUBSPACE SPECTRAL METHODS

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### Abstract

This talk summarizes recent analysis of an alternative approach to the solution of diffusion problems and wave propagation problems in the variable-coefficient case that leads to a new class of numerical methods, called Krylov subspace spectral methods [3].

The basic idea behind these methods, applied to a PDE of the form $du/dt + L(x,D)u = 0$, is to use Gaussian quadrature in the spectral domain to compute Fourier components from elements of $\exp[-L\Delta t]$ for a matrix $L$ discretizing $L(x,D)$ and time step $\Delta t$, using algorithms developed by Golub and Meurant [1], as opposed to applying Gaussian quadrature in the spatial domain as in traditional spectral methods. This strategy allows accurate resolution of all desired components, for both high and low frequencies, without having to resort to smoothing techniques to ensure stability.

This talk focuses on the stability properties of these methods. By describing the Fourier components of the computed solution in terms of directional derivatives of moments, we can demonstrate unconditional stability for parabolic problems, given sufficient smoothness of the coefficients of $L(x,D)$. We also discuss generalizations to systems of equations, including a simple high-order scheme for the second-order wave equation [2]. In this case, we demonstrate that Krylov subspace spectral methods, although they are explicit, are not restricted by the CFL condition.

## References

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[3] J. V. Lambers, *Krylov subspace spectral methods for variable-coefficient initial-boundary value problems*, ETNA, 20 (2005), pp. 212–234.