

# ON OPTIMAL SHORT RECURRENCES FOR GENERATING ORTHOGONAL KRYLOV SUBSPACE BASES

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**Keywords:** Krylov subspace methods, orthogonal bases, short recurrences, conjugate gradient like methods

## Abstract

In this talk I will discuss necessary and sufficient conditions on a nonsingular matrix  $A$ , such that for *any* initial vector  $r_0$ , an orthogonal basis of the Krylov subspaces  $\mathcal{K}_n(A, r_0)$  is generated by a short recurrence. Orthogonality here is meant with respect to some unspecified positive definite inner product. This question is closely related to the question of existence of *optimal* Krylov subspace solvers for linear algebraic systems, where optimal means the smallest possible error in the norm induced by the given inner product. The conditions on  $A$  were first derived and characterized more than 20 years ago by Vance Faber and Tom Manteuffel (SIAM J. Numer. Anal., 21 (1984), pp. 352–362). Their main theorem is often quoted and appears to be widely known. Its details and underlying concepts, however, are quite intricate, with some subtleties not covered in the literature.

The talk will be based on joined work with Zdeněk Strakoš [1], and with Vance Faber and Petr Tichý [2].

**Acknowledgement:** The work was supported by the Emmy Noether Program of the Deutsche Forschungsgemeinschaft (J. Liesen and P. Tichý), and by the Czech National Program of Research “Information Society” under project 1ET400300415 (Z. Strakoš and P. Tichý) and the Institutional Research Plan AV0Z10300504 (Z. Strakoš).

## References

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- [2] V. Faber, J. Liesen and P. Tichý. The Faber-Manteuffel Theorem for linear operators, submitted to SIAM J. Numer. Anal.