MULTILEVEL BDDC IN THEORY AND PRACTICE

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Abstract

The BDDC algorighm has quickly become one of the most popular substructuring methods for symmetric, positive definite variational problems. Like other substructuring method, the method is conjugate gradients preconditioned by independent problems on substructures, solved in parallel, and a coarse problem. For large number of processors, the coarse problem becomes a bottleneck. Therefore, three-level BDDC was proposed, which add another, yet coarser level. Here we extend the three-level BDDC and its theory to an arbitrary number of levels.

We present the formulation and new condition number bounds for a multilevel version of the BDDC algorithm. The condition number bound grows polylogarithmically with the ratio of the substructure sizes between levels, but it is not bounded independently of the number of levels. Numerical results show that this may indeed occur. Since in practice the substructures are large and the number of levels is small, this is acceptable and provides a superb domain decomposition method.

The condition number bounds are based on a new multilevel algebraic theory for nested substructuring methods. The BDDC preconditioner on a finite element space is given by an extension of the variational form of the problem to a direct sum of energy orthogonal spaces and by projections from each of the spaces into the original finite element space. The abstract BDDC preconditioner then consists of solving the variational problem on each space from the direct sum, and adding the solutions projected into the original space. It turns out that both the original BDDC and the multilevel BDDC can be written in this form with a judicious selection of the spaces in the direct sum. Taking maximal advantage of orthogonality is what assures fast convergence of the method, and also leads to an elegant mathematical theory.