LSTRS: MATLAB SOFTWARE FOR LARGE-SCALE TRUST-REGION SUBPROBLEMS AND REGULARIZATION

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Abstract

We describe a MATLAB implementation [6] of the method LSTRS [5] for the large-scale trust-region subproblem:

$$\min \quad \frac{1}{2}x^T Hx + g^T x \ \text{ subject to (s.t.) } \ \|x\|_2 \leq \Delta, \tag{1}$$

where H is an $n \times n$, real, large, symmetric matrix, g is an n-dimensional real vector, and Δ is a positive scalar. Problem (**??**) arises in connection with the trust-region globalization strategy in optimization. A special case of problem (**??**), namely, a least squares problem with a norm constraint, is equivalent to Tikhonov regularization [7] for discrete forms of ill-posed problems.

LSTRS is based on a reformulation of the trust-region subproblem as a parameterized eigenvalue problem, and consists of an iterative procedure that finds the optimal value for the parameter. The adjustment of the parameter requires the solution of a large-scale eigenvalue problem at each step. The method relies on matrix-vector products only and has low and fixed storage requirements, features that make it suitable for large-scale computations. In the MATLAB implementation, the Hessian matrix of the quadratic objective function can be specified either explicitly, or in the form of a matrix-vector multiplication routine. Therefore, the implementation preserves the matrix-free nature of the method. The MATLAB implementation offers several choices for the eigenvalue calculation and it also allows the users to specify their own eigensolver routine.

We present a brief description of the LSTRS method from [5] and describe the main components and features of the MATLAB software. We include comparisons with the following state-of-the-art, large-scale techniques for solving problem (**??**): the Semidefinite Programming approach of Fortin and Wolkowicz [1], the Sequential Subspace Method of Hager [3], and the Generalized Lanczos Trust Region method of Gould et al. [2] as implemented in the HSL library [4]. We present examples of use of the software as well as results from the regularization of large-scale discrete forms of ill-posed problems.

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