RELAXING CONDITIONS FOR THE BRAMBLE PASCIAK CG

Martin Stoll, Andy Wathen
Oxford University Computing Laboratory
Oxford, England
e-mail: martin.stoll@comlab.ox.ac.uk, andy.wathen@comlab.ox.ac.uk

Abstract

In 1988 Bramble and Pasciak introduced a Conjugate Gradient method which is widely used to solve saddle point problems of the form

$$\mathcal{A} = \begin{bmatrix} A & B^T \\ B & -C \end{bmatrix}$$

where the preconditioner

$$P = \begin{bmatrix} A_0 & 0 \\ B & -I \end{bmatrix}$$

is incorporated. The preconditioned matrix $\mathcal{A} = P^{-1}\mathcal{A}$ is symmetric and positive definite in the inner product defined by G. Therefore, the inner product matrix

$$G = \begin{bmatrix} A - A_0 & \\ & I \end{bmatrix}$$

is needed to be symmetric positive definite as does the preconditioner A_0 for A. An improved variant was given by Meyer and Steidten in 2001. Our goal is to relax some of these conditions. We analyze alternative solvers and their behaviour for different situations. We investigate a MINRES implementation with a G-inner product and also the use of the ideal transpose-free Quasi Minimal Residual method proposed by Freund in 1992. Numerical results coming from the IFISS package will be given.