BACKWARD PERTURBATION ANALYSIS OF THE SCALED TOTAL LEAST SQUARES PROBLEM

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Abstract

Given $A \in \mathbb{R}^{m\times n}$ with $m \geq n$, $b \in \mathbb{R}^m$, and $\gamma \in (0, \infty)$, the scaled total least squares (STLS) problem can be defined as follows:

$$\min_{E,f,x}\{\|[E, \gamma f]\|_F\} \quad \text{subject to} \quad (A + E)x = b + f.$$

The STLS problem reduces to the ordinary least squares (LS) and data least squares (DLS) problems as $\gamma \to 0$ and $\gamma \to \infty$, respectively.

Given an approximate solution $y \in \mathbb{R}^n$ to the STLS problem, we derive a formula for a lower bound on the corresponding backward error. When $\gamma \to 0$ the lower bound reduces to the backward error obtained in [2] for the LS problem. When $\gamma \to \infty$ it becomes the lower bound on the backward error of the DLS problem obtained in [1].

Numerical experiments suggest that when y is a reasonable approximation to the exact STLS solution, this lower bound is in fact the backward error.

We also provide an easily computable lower bound and an asymptotic estimate for the above lower bound, and discuss how these could be useful for the design of stopping criteria for iterative solution methods.

References

[1] X.-W. Chang, G. H. Golub, and C. C. Paige, *Backward perturbation analysis for data least squares problems*, submitted for publication.

[2] B. Waldén, R. Karlson, and J.-G. Sun, *Optimal backward perturbation bounds for the linear least squares problem*, Numerical Linear Algebra with Applications, 2 (1995), pp. 271–286.