Solving Regularized Total Least Squares Problems

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Many problems in data estimation are governed by overdetermined linear systems

$$Ax \approx b, \quad A \in \mathbb{R}^{m\times n},\ b \in \mathbb{R}^m, m \geq n \tag{1}$$

where both the matrix A and the right hand side b are contaminated by some noise. This may happen for instance if the matrix A is obtained by measurements.

A possible approach to this problem is the total least squares (TLS) method which determines perturbations $\Delta A \in \mathbb{R}^{m\times n}$ to the coefficient matrix and $\Delta b \in \mathbb{R}^m$ to the vector b such that

$$\|[\Delta A, \Delta b]\|_F^2 = \min! \quad \text{subject to } (A + \Delta A)x = b + \Delta b \tag{2}$$

where $\|\cdot\|_F$ denotes the Frobenius norm of a matrix.

In practical situations the linear system is often ill-conditioned. In these cases the TLS solution can be physically meaningless, and regularization is necessary for stabilizing the solution.

In this talk we consider regularization by adding a quadratic constraint, and we discuss methods for regularized total least squares problems which are based on sequences of eigenvalue problems. Methods of this type were considered by Renaut and Guo [**?**], and by Sima, Van Huffel and Golub [**?**]. Both types of methods can be improved considerably taking advantage of thick initial bases and early updates [**?**, **?**].

References

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