STRUCTURED MATRIX METHODS FOR THE COMPUTATION OF MULTIPLE ROOTS OF UNIVARIATE POLYNOMIALS

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Abstract

Significant effort has been devoted to the development of numerically reliable algorithms for the computation of the roots of a polynomial, but problems persist. For example, computations with the Wilkinson polynomial show that the accurate determination of the simple roots of a polynomial may not be trivial. Furthermore, these problems are compounded as the degree of the polynomial increases and/or the polynomial has multiple roots.

This paper describes a radically new method for the computation of the roots of a univariate polynomial. In particular, computations that involve the Sylvester resultant matrix of the polynomial and its derivative are used to calculate the multiplicities of the roots. The Sylvester matrix has a well-defined structure, and thus structured matrix methods are used to obtain improved results to the resulting least squares equality problem. The multiplicities are then used as constraints when the roots are calculated, and the method of non-linear least squares is used to improve the initial estimates of the roots.

Interesting problems occur when the coefficients of the polynomials are inexact, in which case there exists a (potentially infinite) family of solutions. In this case, it may be required to compute the roots that have a particular multiplicity structure, and it is therefore necessary to compute the smallest perturbation of the given inexact polynomial, such that the roots of the perturbed polynomial satisfy the given constraints on their multiplicities. These computations are very similar to those that are required for the computation of an approximate greatest common divisor of two inexact polynomials. The algorithm requires an estimate the rank of a noisy matrix, and it is shown that the *minimum description length* (MDL) can be used to obtain this quantity. The fundamental idea of the principle of MDL, which is closely related to Occam's razor, is that any regularity that is present in a given set of data can be used to compress the data. Since it is desired to select the hypothesis that captures the most regularity in the data, the hypothesis that achieves maximum compression is selected.

The talk will contain several computational examples in order to illustrate the theoretical ideas.