INTERVALS, TRIDIAGONAL MATRICES AND THE LANCZOS METHOD

Wolfgang Wülling
Basler Versicherungen,
Aeschengraben 21, Postfach,
CH - 4002 Basel, Switzerland,
e-mail: wolfgang.wuelling@baloise.ch

Keywords: Lanczos method, tridiagonal matrix, eigenvalues, Ritz values, Riemann-Stieltjes integral, Gauss approximations

Abstract

Consider the eigenvalue problem $Au = \lambda u$, where A is a large, real symmetric and sparse matrix. The Lanczos method (or any of its variants) is the method of choice if one is to seek approximations to a few eigenpairs (λ, u) of A. Due to rounding errors in finite precision computations the Lanczos vectors may lose their orthogonality even for a small number of iterations. Consequently, it can be numerically observed that clusters of Ritz values are generated: multiple copies of Ritz values all approximating a single eigenvalue of A.

In this contribution we will work on two conjectures posed by Zdeněk Strakoš and Anne Greenbaum in [1] on the clustering of Ritz values.

The first conjecture is as follows: Has any Ritz value in a tight, well separated cluster stabilized to within a small quantity δ which is proportional to the square root of the length of the cluster interval and the gap in the spectrum?

The second conjecture is concerned with the concept of *stabilization of weights*. Any nonzero vector together with the eigenvectors and eigenvalues of A defines a weight function and hence a Riemann-Stieltjes integral. The Lanczos method implicitly computes a sequence of Gauss quadrature approximations. This situation leads to the following question: If a cluster of Ritz values closely approximates an eigenvalue λ of A, does the sum of weights corresponding to the clustered Ritz values closely approximate the original weight belonging to the eigenvalue λ?

We will briefly present our solutions in this talk.

References

[1] Z. Strakoš and A. Greenbaum, *Open questions in the convergence analysis of the Lanczos process for the real symmetric eigenvalue problem*, IMA Research Report 934, IMA Workshop on Iterative Methods, University of Minnesota, Minneapolis, MN, 1992.