MATRIX SECTOR FUNCTION – PROPERTIES AND ALGORITHMS

Beata Laszkiewicz, Krystyna Ziętak
Institute of Mathematics and Computer Science,
Wrocław University of Technology,
50-370 Wrocław, Poland,
e-mail: beata.laszkiewicz@pwr.wroc.pl,
krystyna.zietak@pwr.wroc.pl

Keywords: matrix sector function, Newton's and Halley's methods, Padé approximation

Abstract

Matrix functions arise in numerous applications in science and engineering. This talk is concerned with the matrix sector function, introduced by Shieh, Tsay and Wang as a generalization of the matrix sign function (see L.S. Shieh, Y.T. Tsay, C.T. Wang, Matrix sector functions and their applications to system theory, IEEE Proceedings 131(5): 171–181, 1984.). Our goal is to review properties and applications of the matrix sector function, and analyze some known algorithms.

For a positive integer p and a matrix $A \in \mathbb{C}^{n\times n}$ having no eigenvalues with argument $(2k+1)\pi/p$ for $k = 0, 1, \ldots, p-1$, the matrix sector function is defined by $\mathrm{sect}_p(A) = A(\sqrt[p]{A^p})^{-1}$, where $\sqrt[p]{X}$ denotes the principal pth root of X. If $A = Z\mathrm{diag}(J_1, \ldots, J_m)Z^{-1}$ is a Jordan cannonical form, where J_l is the Jordan block of order r_l corresponding to the eigenvalue λ_l, then

$$\mathrm{sect}_p(A) = Z\mathrm{diag}(s_p(\lambda_1)I_{r_1}, \ldots, s_p(\lambda_m)I_{r_m})Z^{-1},$$

where $s_p(\lambda)$ is the scalar sector function. Let

$$\lambda = |\lambda|e^{\theta i} \in \mathbb{C}, \qquad \theta \in \left(\frac{2\pi}{p}(k-\frac{1}{2}), \frac{2\pi}{p}(k+\frac{1}{2})\right) \qquad k = 0, 1, \ldots, p-1.$$

Then $s_p(\lambda) = e^{i2\pi k/p}$. The matrix sign function is a particular case for $p = 2$.

Although the matrix sector function can be computed directly from the spectral decomposition of A, an alternative approach based on iterative methods is useful. Numerical experiments comparing Newton's and Halley's methods, and the Padé recursions for computing the matrix sector function are presented. Our attention is drawn to the convergence and stability of the algorithms. We also deal with conditioning of the matrix sector function.