Modelling and Simulation of Complex GeoTechnical Problems

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1 Introduction

This paper provides three examples of topical geotechnical problems which require application of mathematical modelling and use of efficient numerical methods and powerful parallel computers. These examples give a motivation and exploit methods developed within the project MSTEP: Modelling and Simulation of complex Technical Problems.

2 Stability of mine openings

An extensive finite element analysis of stability of mining openings at the uranium mine GEAM D. Rožínka was firstly performed in the years 1996-2002, see Fig. 1. The final conclusions of this analysis advocate the use of a new mining technology (underhand stoping) and safety of mining in greater depths 950-1100 meters under the surface. At present, due to the price increase of the uranium, the GEAM mine prolonged its production life. New plans assume mining at depths up to 1200 meters, see [10]. After a detailed exploration at deeper levels and design of a new production project a new extensive finite element modelling is expected to be performed in the years 2009-2010.

More details concerning the past finite element modelling for GEAM can be found in [5]. Let us summarize the main features taken into account for the construction of the numerical model:

- 3D domain of interest with dimensions $1430 \times 550 \times 600$ metres,
- the uranium ore is located in several meters thick veins coupled in zones of thickness from 4 to 20 meters, see Fig. 1,
- according to the initial stress measurement by the hydrofracturing method, the mechanical loading is given by horizontally anisotropic initial stress and weight of rocks,
- the stability assessment needs to model the advance of mining.

The finite element analysis then

- uses a basic global discretization with linear triangular finite elements and grid with $124 \times 137 \times 76$ nodes,
- assumes linear elastic behaviour of all materials (ore, surrounding rocks, crushed goaf material) and elastic model with nearly 4 million DOFs,
- meets with a big difference between the scale of the whole domain (hundreds of meters) and dimensions of veins and mine openings (metres). It requires highly refined mesh and/or some multiscale approach (in our case the composite grid method),
- simulates the loading by a general anisotropic initial stress by using pure force boundary condition. If additionally, different weights of rocks are considered, then analysis leads to a singular inconsistent linear algebraic system and seeking for its generalized solution,

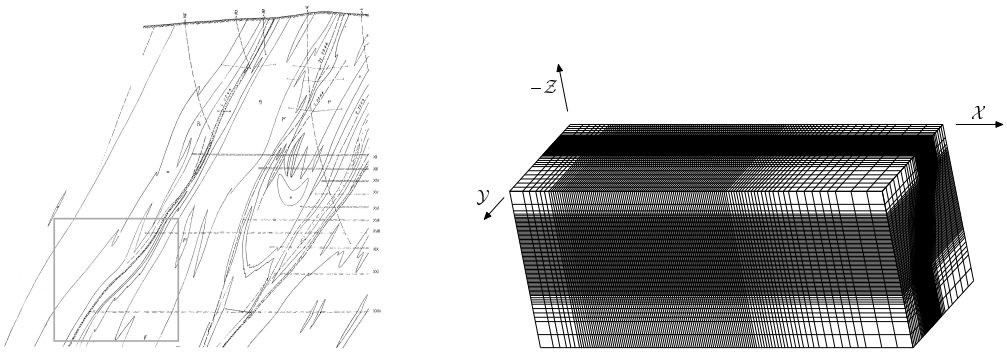

Figure 1: Cross section through Rožná deposit and the computational grid.

- simulates the advance of mining by solving several stages which differs by different inner structure (mined out volumes) inside the domain.

The numerical solution is performed with the use of an in-house FE software GEM, which implements the following numerical methods including a black box two level Schwarz preconditioning method [5]:

- iterative solution of the large scale linear system by CG method,
- use of parallel computation of the iterations with the aid of data partition corresponding to a domain decomposition (DD),
- preconditioning the CG method by Schwarz type overlapping DD method with subdomain subproblems solved approximately by incomplete factorization,
- improved Schwarz preconditioning by adding a global coarse problem in an additive or multiplicative way. The coarse problem is created algebraically by using aggregations,
- approximate solution of the coarse problem by inner CG iterations preconditioned by incomplete factorization,
- stabilization of the outer CG by explicit orthogonalization of the search directions, see [1]. It enables to use inner CG iterations as well as nonsymmetric multiplicative combinations of overlapping Schwarz preconditioner with coarse problem solution,
- stabilization of the iterative solution of singular system by deflation [3].

3 Underground deposition of spent nuclear fuel

The problem of underground deposition of the spent nuclear fuel differs from the previous stability problem in two main aspects. In the Czech Republic, this project is still not located to a specific site and its construction is planned to a relative far future of about 2060. But the selection of site and preparation of the construction concept need more and more detailed knowledge about the repository performance, which is gradually obtained through a generic reference project, see e.g. [11], and mathematical modelling of selected features. Moreover, whereas the modelling of previous section was restricted to mechanical behaviour of rocks, the assessment of nuclear waste repository include mechanical behaviour, thermal loading and groundwater flow with transport of species, especially of radionuclides. Thus the mathematical analysis consider thermo-hydro-mechanical processes (T-H-M) plus chemistry (C), which become especially important due to very long time periods of tens or hundreds thousands years for which

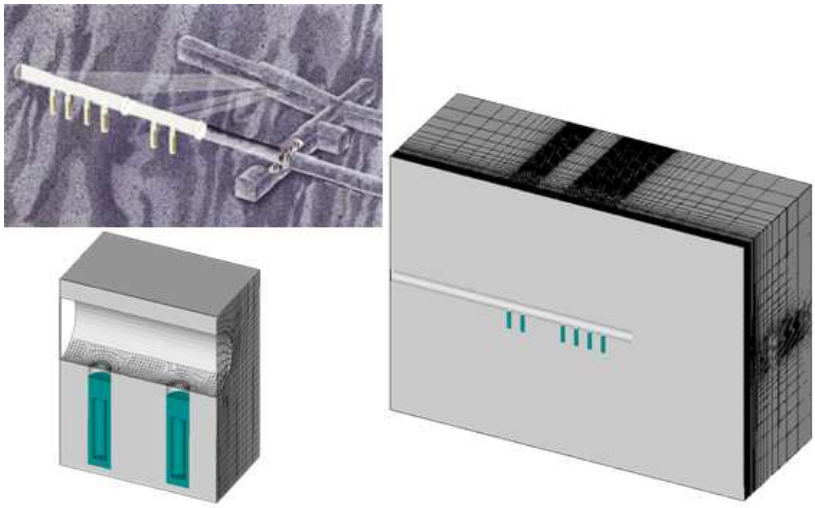

Figure 2: Äspö prototype repository: the considered domain and detail with two deposition holes. Discretization uses a regular grid.

the repository should operate as a barrier between the radioactive waste and the environment. In many cases, the T-H-M-C processes should be considered as coupled [7].

A simple example of problems arising from mathematical simulation of performance of underground nuclear waste repositories is modelling of thermoelastic aspects of the prototype repository experiment in the underground hard rock laboratory in Äspö in Sweden, see Fig. 2. The basic features of this modelling are the following:

- 3D domain of interest has dimensions $200 \times 100 \times 200$ metres, this domain contains tunnel with six deposition holes equipped with canisters (heaters),
- the mechanical loading is given by the weight of rocks and the overburden,
- thermoelastic behaviour is considered in the period of 50 years after installation of canisters.

The finite element analysis uses

- discretization with linear triangular finite elements and with a regular grid containing $391 \times 63 \times 105 = 2\,586\,465$ nodes. It means that more than 2.5 million DOFs is used for heat conduction and more than 7.75 million DOFs for elasticity,
- the FE grid which again reflects a big difference between the scale of the whole domain (hundreds of meters) and dimensions of canisters, deposition holes and pillars (metres). It requires highly refined mesh or multiscale approach.

The problem is again solved by the in-house GEM software, which implements the following numerical methods, cf. [6]:

- time discretization by a backward Euler method, which guarantees full stability without any oscillations of the solution,
- time step determination by adaptive scheme based on comparison of first and second order approximation in time,

- iterative solution of large scale linear systems by CG method and parallel computation of the iterations with the aid of data partition corresponding to domain decomposition,
- preconditioning of the CG method by Schwarz type DD method, the subdomain subproblems are solved approximately by incomplete factorization,
- improved preconditioning of elasticity part by adding a coarse problem created algebraically by aggregations. It is important, that in the heat evolution part, this addition is not necessary, see [5].

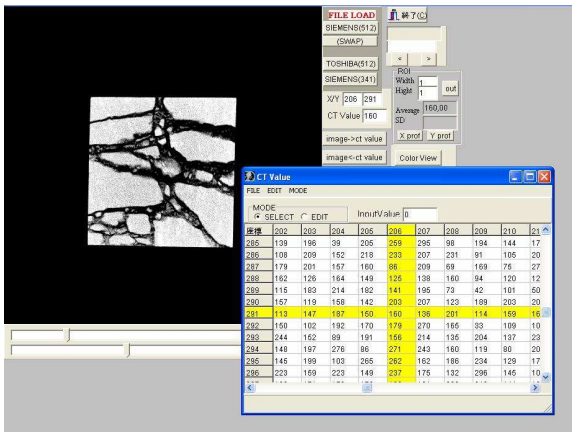

Figure 3: A coal geocomposite (coal and polyurethane resin) - 2D cut and CT values in a selected voxel and its neighbours.

4 Microstructure modelling

The H-M processes in geomaterials depend heavily on the microstructure. The FE analysis of the microstructure can be used for computation of homogenized material parameters, developing constitutive relations for poroelasticity, clearing up the coupling effects etc.

A specific application of microstructure FEM (μFEM) is an analysis of properties of geocomposites, see Fig. 3. This analysis allows to compute homogenized characteristics as well as to optimize the properties of polyurethane resin filled into the geomaterial (coal) matrix. Specific features of this problem are:

- analysis of 3D cube with edge 75mm discretized with an uniform voxel grid with $251 \times 251 \times 76 = 4\,788\,076$ nodes and nearly 15 million DOFs in the case of elasticity,
- use of uniform grid with assuming homogeneous material in voxels. Material properties of voxels are assigned according to X-ray CT scan.

More details can be found in [9] and in a future paper.

5 Conclusion

Paper shows three examples of problems from geotechnics, which lead to the solution of large scale linear systems. Schwarz type domain decomposition technique is used for parallel solution of systems arising from standard FEM [5, 6] as well as mixed FEM [4, 8]. A tutorial lecture on parallel CG with Schwarz type preconditioning was given at SNA 05, see [2]. The importance of efficient solvers can further strengthened by solving nonlinear and coupled problems and by attempts to evaluate influence of uncertainty in model definition.

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