Mathematical Modelling of an Underground Repository of the Spent Nuclear Fuel

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Introduction

The paper concerns the mathematical modelling of thermo-mechanical processes in nuclear waste repositories. The model applies only the one-directional thermo-mechanical coupling via thermal expansion term in the constitutive relations. Thus, the thermo-elasticity problem can be divided into two parts. Firstly, the temperature distribution is determined by the solution of the nonstationary heat conduction equation, and secondly, the linear elasticity problem is solved at given time points. The initial-boundary value problem of thermo-elasticity is discretized by the linear tetrahedral finite elements in space and by the simplest finite differences in time. In this paper we consider only the thermal part of the problem which leads to the repeated solution of the linear system

$$B_h \Delta \underline{\tau}^j = (M_h + \Delta_j K_h) \Delta \underline{\tau}^j = f_j$$

in each time step. For the solution of the linear systems we use the preconditioned CG method where the preconditioning is given by the additive overlapping Schwarz method. The linear systems are solved in parallel.

The aim of the contribution is to show the experience with the computing of a large practical problem concerning the assessment of the influence of the distance between the deposition places to the thermo-elastic response of the rock massif.

Model example

The model example comes out from the depository design proposed in [2]. The whole depository is very large. For the FE model of the whole depository it would be necessary to use the large domain with several parallel panels each containing a central tunnel area and tens of tunnel pairs. Totally there are thousands of canisters. But using symmetry we can solve the problem only on the part of the domain (see Figure 1).

The question is how the results obtained by the numerical solution on a part of the domain correspond to the solution on the whole domain. We use two simplified test tasks with rectangular FE grids. The large task corresponds to one panel with the central tunel area and 7 tunel pairs, each with 14 canisters, which is situated in the large domain ($1129 \times 1190 \times 1127m$). The boundary of domain is about 300m far of the panel not to influence results. The small task ($31.2 \times 58.0 \times 107m$) contains three depository tunnels (a half of the tunnels) , each with four deposition holes and one access tunnel. The tunnels are filled with a backfill. This small task is the rectangular version of the task presented in Figure 1. We suppose zero initial temperature in the area and compute the temperature field initiated only by the heating from the radioactive waste. We follow the temperature at eight points: A - the centre of the canister, B - the centre point between the tunnels, C - the point above the tunnel (the distance 10.2m from the canister),

D - the point below the bottom of the deposition hole (the distance 1.9m from the canister), E - the middle point between two canisters (the distance 2.8m from the canister), F - the point above tunnel (the distance 17.7m from the canister), G - the point above tunnel (the distance 26.7m from the canister), H - the point above tunnel (the distance 39.2m from the canister - the upper boundary of the subdomain). The temperatures in these points for global and local tasks at the time point $t = 6.66$ years (the maximum reached temperature) are shown in the Table 1. The Table shows that the results obtained from the task on subdomain are

	A	*B*	C	D	E	F	G	H
global task	114.89	19.56	15.74	43.71	47.93	8.24	3.60	0.96
local task	114.97	19.64	15.84	43.79	47.99	8.36	3.85	1.95

Table 1: The temperatures in the given points - the comparison between the global and the local tasks.

very close to the global results. The biggest difference is on the upper boundary but there is the influence of the canisters on the temperature field very small. So the solution on the subdomain is very good approximation of the global solution in the neighbourhood of the deposition holes.

Now we solve the problem on the subdomain using nonrectangular grid. We test four variants with the distances between the holes from 2.68m to 7.5m. The whole model domain is situated 800m under surface. A constructed 3D T-M model of repository is shown in Figure 1.

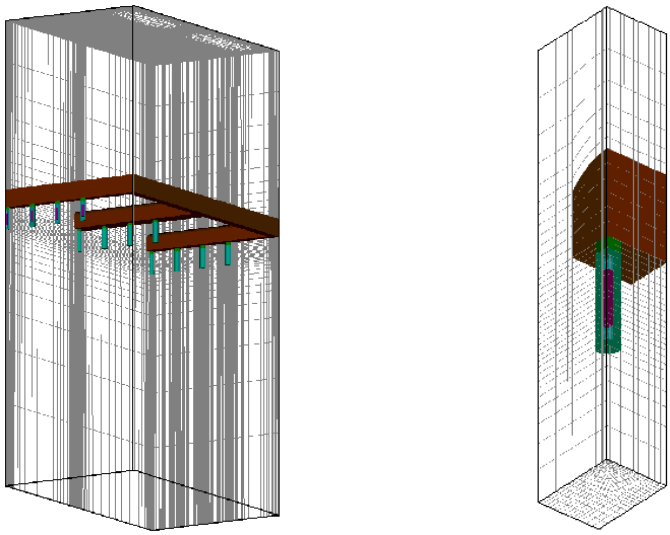

Figure 1: Finite element mesh for repository model.

The computations are done in four subsequent phases (including mechanical part):

- the phase of virgin rocks - the initial stresses are determined from the weights of the rocks, the initial temperature is determined using geothermal gradient
- the tunnels are excavated. The elasticity problem is solved using equivalent forces on the faces of tunnels initiated by the excavation. The nonstationary heat problem is solved for period 10 years with the initial condition determined in the phase 1 and with the heat transfer on the faces of the tunnels

- the deposition holes are excavated. The elasticity problem is solved using equivalent forces on the faces of the holes
- the thermoelasticity problem is solved for the period 150 years with the initial condition given by the temperature computed in the phase 2.

Numerical experiments

In each time step we solve the system of linear equations with PCG. The in-house solver GEM is working in single precision (real*4) and using stopping criterion based on the norm of the residual vector. In Figure 2a we show the behaviour of the residual vector for solving the system of linear equations in the case of stationary heat equations for real*4 and real*8 where the residual vector is given both by the recurent formula ($R^k_{rec} = r_k = r_{k-1} + ...$) and by the "exact" computing ($R^k_{ex} = r_k = Ax_k - f$). We can see that in real*4 $\|R^k_{ex}\|$ decreases till the value 5.0×10^{-3} while $\|R^k_{rec}\|$ shows incorrectly the permanent convergence. In real*8 the norm of the "exact" residual vector shows convergence till the value 2.0×10^{-11}. With respect to these results it's clear that using the value $\varepsilon < 5.0 \times 10^{-3}$ for the stopping criterion is useless in our task.

The question is whether the value $\varepsilon = 5.0 \times 10^{-3}$ is sufficient for the accuracy of the solution. The figure 2b shows the behaviour of $\|R^k_{ex}\|_{l_2}$, $\|R^k_{ex}\|_{C^{-1}}$, $\|R^k_{ex}\|_A$ and $\|u - u_{ex}\|_{l_2}$ (logaritmic scale) during iterations. We can see that the error of the solution corresponding to the value $\|R^k_{ex}\|_{l_2} = 5.0 \times 10^{-3}$ is approximately 5.0×10^{-5} (in l_2 norm). We can also see, that $\|R_{ex}\|_{C^{-1}}$ gives better approximation of the error than $\|R_{ex}\|_{l_2}$. From the practical point of view we are

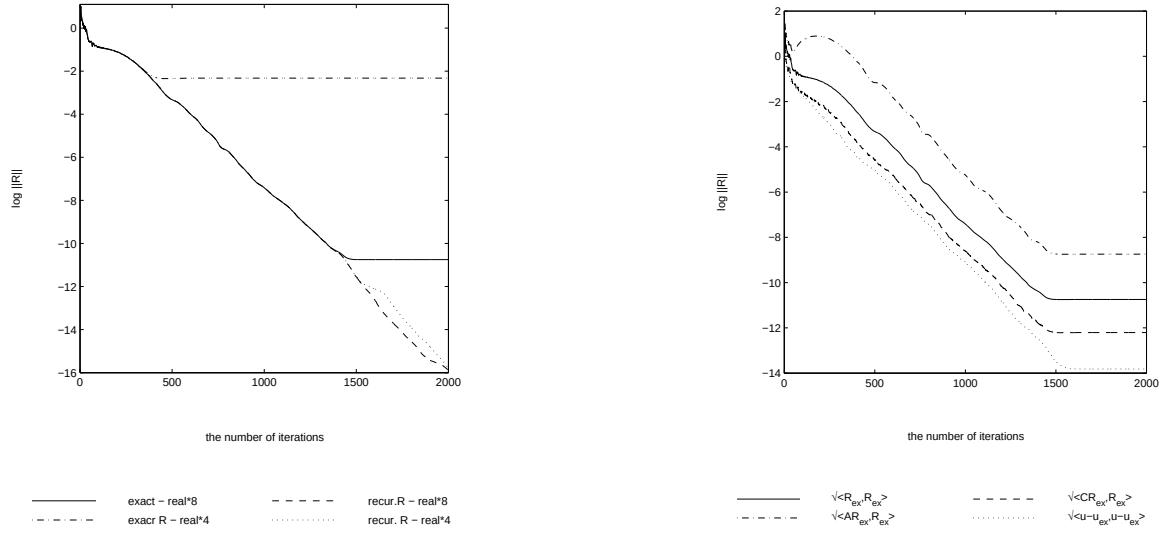

Figure 2: a) (left) The behaviour of $\|R^k_{ex}\|_{l_2}$ for real*4, real*8, "exact" R, "recurent" R, b)(right) the behaviour of $\|R^k\|$ for various norms, the behaviour of the error.

interested in the nodal errors (differences in the nodal temperatures). If we compare the solution obtained with the solver both in single precision ($\|R_{ex}\|_{l_2} = 5.0 \times 10^{-3}$) and in double precision ($\|R_{ex}\|_{l_2} = 2.0 \times 10^{-11}$), the maximum difference of the nodal temperatures is about 1.0×10^{-3}, so the solver in single precision is good enough for the solution of the problem. How the choice of ε influences the global solution of the nonstationary problem we can see in Table 2. The

ε	0.5	0.1	0.01	0.001
nbr of time steps	456	169	71	70
global nodal error	1.86	1.17	0.038	0.0019
nbr of PCG iterations	302	543	1200	1987
CPU time (s)	7908	5190	9763	15508
nodal err for the stationary case	0.620	0.097	0.014	0.0011

Table 2: The results of computing for various values of ε.

smaller value of ε reduces the number of time steps, but larger time step gives bigger error in the time discretization. Therefore even $\varepsilon = 0.5$ gives relatively good approximation of the global nonstationary solution. The last row shows the nodal error for the solution of the stationary heat problem. The graphics results of computing for the various distances of deposition holes are presented in Figure 3(more see [1]).

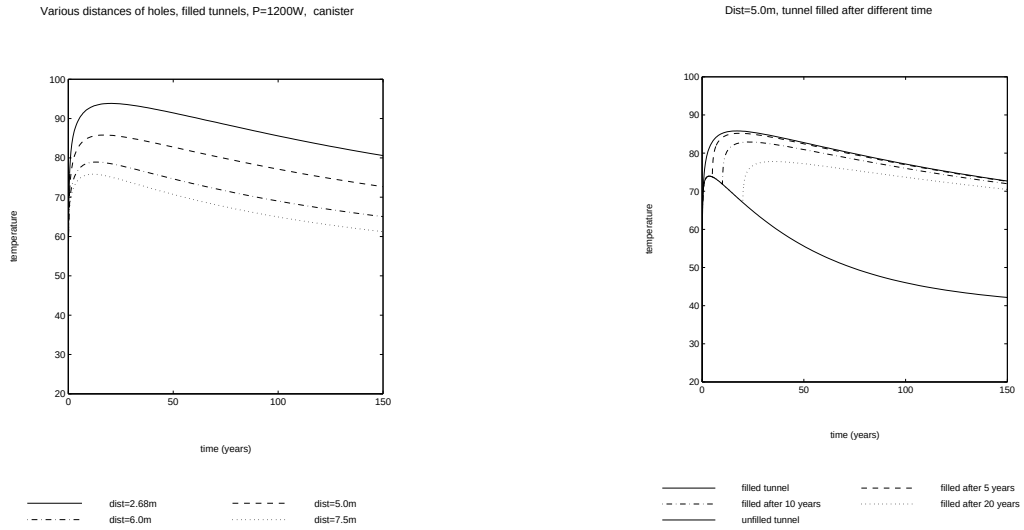

Figure 3: The dependence of temperature on time: a) various distances of holes, b) the filling of tunnels in different time.

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References

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