Simpler GMRES or GCR?

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In this contribution we analyze the numerical behavior of several minimum residual methods, which are mathematically equivalent to the GMRES method. Two main approaches are compared: the one that computes the approximate solution (similar to GMRES) in terms of a Krylov space basis from an upper triangular linear system for the coordinates, and the one where the approximate solutions are updated with a simple recursion formula. We show that a different choice of the basis can significantly influence the numerical behavior of the resulting implementation. While Simpler GMRES [2] and ORTHODIR [4] are less stable due to the ill-conditioning of the basis used, the residual basis is well-conditioned as long as we have a reasonable residual norm decrease. These results lead to a new implementation, which is conditionally backward stable, and, in a sense they explain the experimentally observed fact that the GCR [3] (also known as ORTHOMIN [4] or GMRESR [5]) method delivers very accurate approximate solutions when it converges fast enough without stagnation.

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