# On the solution of the linear systems arising in the Simplex method with modern linear solvers

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The simplex method ranks among the most popular methods for linear programs (LP's), especially for mixed integer programs. In most existing Simplex-based LP codes, direct solution of the arising linear systems via dynamic Markowitz pivoting, originally described in the 1957 paper [1], is used. In light of the active development of direct as well as iterative solvers in the numerical linear algebra community over the last decades, it seems surprising that most LP codes are still based on a technique that dates back that long. This contribution investigates the appropriateness of several popular modern linear solvers to solve the linear systems arising in the simplex method.

In the simplex method system matrices are in general very large and sparse and differ from iteration to iteration by one sparse column only. The involved matrices are non-symmetric and indefinite and seem, at first sight, to lack any structure. We will show, however, that it is possible to find row and column permutations that reveal a pronounced structure which is typical for system matrices in the simplex method. They enable to neglect a very large part of the linear system so that one can concentrate on a system of a dimension which is frequently only one percent of the original dimension. We will concentrate on linear solvers for the remaining small systems and show that, in the vast majority of cases, iterative solvers like preconditioned Krylov subspace methods are not well suited to solve the these systems. As for direct solvers, important differences in performance can be observed. We will compare a representative set of modern direct solvers and demonstrate that, for these specific linear systems, more modern solvers are often less efficient in that they produce a fill-in of several times the fill-in with dynamic Markowitz pivoting.

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## References

[1] Markowitz, H.M.: *The elimination form of the inverse and its application to linear programming*, Management Sci. 3, 255269 (1957).