A comparison of direct linear solvers for the solution of the linear systems arising in the Simplex method.

R. Luce, J. Duintjer Tebbens, J. Liesen and R. Nabben

Institute of Mathematics, Technical University of Berlin

One of the most popular methods for linear programs (LP's) is the simplex method. In most existing simplex-based LP codes, the arising linear systems are solved directly via dynamic Markowitz pivoting, originally described in the 1957 paper [1]. It seems surprising that most LP codes are still based on a technique that dates back that long. This contribution investigates the appropriateness of several popular modern direct solvers to solve the linear systems arising in the simplex method. It presents and comments the results with different software packages of numerical experiments on a large number of benchmark LP's.

In the simplex method system matrices are in general very large and sparse and differ from iteration to iteration by one column only. The involved matrices are non-symmetric and indefinite and seem, at first sight, to lack any structure. We will show, however, that it is possible to find row and column permutations that reveal a pronounced structure which is typical for system matrices in the simplex method. They enable to neglect a very large part of the linear system so that one can concentrate on a system of a dimension which is, as we demonstrate, frequently only one percent of the original dimension. We first explain why using some modern direct solvers without the mentioned row and column permutations leads to an unacceptable amount of fill-in and then concentrate on the solution of the remaining small systems. We will compare a representative set of modern direct solvers and show that here too, more modern solvers are often less efficient in that they produce a fill-in of several times the fill-in with dynamic Markowitz pivoting.

References

[1] Markowitz, H.M.: *The elimination form of the inverse and its application to linear programming*, Management Sci. 3, 255269 (1957).