GMRES Acceleration Analysis for a Convection Diffusion Model Problem

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Streamline upwind Petrov Galerkin discretization on an $N \times N$ grid of the convection diffusion model problem

$$\nu\nabla^2 u + w \cdot \nabla u = f \text{ in } \quad \Omega = (0,1) \times (0,1),$$

$$u = g \text{ on } \partial\Omega,$$

where $w = [0,1]^T$ denotes a vertical wind, yields a stiffness matrix A that can be reduced to a simple form via orthogonal similarity transformation - a block diagonal form with nonsymmetric tridiagonal Toeplitz $N \times N$ blocks. As has been proved by Liesen and Strakoš, during the initial phase of GMRES processes, that is untill the Nth iteration, convergence behaviour is governed by the Toeplitz block problem exhibiting the slowest convergence speed. However, no theoretical description of the second phase of convergence was known so far. In this talk we precisely focuss on convergence behaviour after the Nth iteration. In particular, we wish to quantify the sudden acceleration of GMRES around the Nth step observed in our experiments. We will use constant diagonal translation

$$B = A + \lambda I,$$

for some parameter λ and demonstrate that with an appropriate combination of scaling and diagonal translation some properties of the matrix

$$[r_0, Br_0, \ldots, B^k r_0],$$

where r_0 denotes the initial residual, can drastically change at the step N. The latter fact can be formalized in lower and upper bounds for the GMRES residual. We demonstrate the accuracy of the bounds on examples.
This work was supported by the project 1ET400300415 within the National Program of Research Information Society.

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