Analysis of the total least squares problem with multiple right-hand sides

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Let A be a real m by n matrix and b a real vector of the lenght m. In *C. C. Paige, Z. Strakoš: "Core problems in linear algebraic systems", SIAM J. Matrix Anal. Appl. 27, pp. 861–875 (2006)*, it is proved that for any orthogonally invariant linear approximation problem $Ax \approx b$, the partial Golub-Kahan bidiagonalization of the matrix $[b, A]$ determines a minimally dimensioned *core approximation problem* $A_{11}x_1 \approx b_1$ containing the necessary and sufficient information for solving the original problem. It is shown how the core problem can be used in a simple and efficient way for solving the total least squares and the data least squares formulation of the original approximation problem.

In this contribution we concentrate on a possible generalization of the core problem idea to total least squares problems $AX \approx B$ with *multiple right-hand sides*. Here the existence of $d > 1$ simultaneously approximated columns of the matrix B can lead to significant complications. We investigate a reduction of the original data, and solvability of the resulting reduced problem. Difficulties with the definition of the solution in the total least squares formulation will be illustrated on examples.