Lanczos tridiagonalization and the core problem

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The Lanczos tridiagonalization is a well known algorithm that transforms a real symmetric matrix $\mathbf{A}$ to a symmetric tridiagonal form. The Golub-Kahan bidiagonalization, also referenced as the Lanczos bidiagonalization, reduces a general rectangular matrix to upper or lower bidiagonal form. There are several ways describing the relationship between these two algorithms, see, e.g., C.C. Paige: Bidiagonalization of matrices and solution of linear equations in SIAM J. Numer. Anal. 11 (1974).

In the paper Core problems in linear algebraic systems (to appear in SIAM J. Matrix Anal. Appl.), C.C. Paige and Z. Strakoš analyze a linear approximation problem $Ax \approx b$. They describe the following fundamental property of the bidiagonalization algorithm: when it is applied to an extended matrix $[b, A]$, it determines the so called core problem, which extracts all necessary and sufficient information needed to solve the problem with the original data. In this contribution we explain the relationship between the core problem theory and the known results about the Lanczos tridiagonalization.