On fundamentals of total least squares problems

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Consider an overdetermined linear approximation problem $Ax \approx b$, where A is a real m by n matrix, b is a real m-vector. In *total least squares* (TLS) this problem is solved by constructing minimal correction to the vector b and the matrix A such that the corrected system is compatible,

$$\min_{g,E,x} \|[g,E]\|_F \quad \text{subject to} \quad (A+E)\,x \;=\; b+g\,, \tag{1}$$

see [3, 4]. If the TLS solution exists and it is unique, then it can be found from the scaled right singular vector of the matrix $[b, A]$ corresponding to its smallest singular value. Therefore TLS can be alternatively formulated as

$$\min_{z=(-1,x^T)^T} \frac{\|\,[b,A]z\,\|_2^2}{\|z\|_2^2} \;=\; \min_{x} \frac{\|\,b-Ax\,\|_2^2}{1\,+\,\|x\|_2^2}\,; \tag{2}$$

some recent applications of (2) can be found in [1, 7]. For more general optimization formulations of TLS-related problems we refer to [2, 3, 4, 8].

Contrary to the standard least squares approximation problem, the (finite) solution of (1), (2) does not always exist. That means that the TLS problem above is not for some data A, b correctly defined. In this contribution we discuss the necessary and sufficient condition for the existence of the TLS solution based on the so called *core problem* theory [5], and mention work on possible extensions of the analysis to the multiple right hand sides case $AX \approx B$, where B is a real m by d matrix [6].

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