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An Updated Preconditioner for Sequences of General Nonsymmetric Linear Systems

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Efficient solution of *sequences* of linear systems is a task arising in numerous applications in engineering and scientific computing. Some examples are simulation of processes in fluid dynamics where every time step requires the solution of a system of nonlinear equations, operation research problems where the linear programs are solved with the simplex method or solution of Helmholtz equations for the propagation of time-harmonic waves. Depending on the linear solver and the properties of the system matrices, several techniques to share part of the computational effort throughout the sequence may be used. Apart from the very simple idea of using hot starts with the solution of the previous system, in some cases exact updating of the system matrix is feasible even for large problems. Rank-one updates of LU factorizations have been used since decades in the simplex method where the change of one system matrix to another is restricted to one column [17]. General rank-one updates of an LU decomposition are discussed in [16]. If the linear solver is a Krylov subspace method, strategies to recycle information gained from previously generated subspaces have shown to be beneficial in many applications [10, 13]. In the special case where shifted linear systems with identical right hand sides have to be solved, Krylov subspace methods allow advantageous implementations based on the fact that all systems generate the same subspace [8]. In nonlinear systems solved with a Newton-type method a well-known strategy is to skip evaluations of the (approximate) Jacobian during some iterations, leading to Shamanskii's combination of the chord and Newton method [5], [14]. Several subsequent systems of the sequence then differ only by their right hand side and linear solving techniques with multiple right hand sides can be exploited [15]. Another way to save costs is to allow changing the system matrices but freezing the preconditioner [9]. To enhance the power of a frozen preconditioners one may also use *approximate* updates; in [11] we find approximate updates of incomplete Cholesky factorizations and in [1, 3] banded updates were proposed for both symmetric positive definite approximate inverse and incomplete Cholesky preconditioners. In Quasi-Newton methods the difference between system matrices is of small rank and preconditioners may be efficiently adapted with approximate small-rank updates; this has been done in the symmetric positive definite case, see [2, 12].

Our contribution considers a new black-box approximate update scheme for factorized preconditioners introduced in [6] and extended in [4, 7]. It is designed for general nonsymmetric linear systems solved by arbitrary iterative solvers and hence it can be combined with some of the techniques for more specific systems and solvers mentioned before. The basic idea is to combine an incomplete reference factorization with a Gauss-Seidel type of approximation of the difference between the current and the reference matrix. The technique tends to correct deteriorating numbers of iterations needed to solve with a frozen preconditioner by reducing them to an acceptable level. Moreover, the updated factorizations can be more powerful than preconditioners computed from scratch; this may happen, for instance, when the updates are related to more stable reference preconditioners generated earlier in the sequence. Since the updated factorizations are both cheap to compute and cheaply applied as a preconditioner, their use is of considerable interest for practice. This is especially true when preconditioner computations are expensive, like in matrix-free environment where the matrix has to be estimated first. In this talk we give a brief description of the basis update technique and then we present new theoretical results on the quality of the updates. Apart from providing new insight they may serve as guidelines in deciding whether to switch-on the usage of updates in black-box solvers for sequences of linear systems. We also present numerical experiments with a new implementation strategy for matrix-free environment which is based on mixed matrix-free/explicit triangular solves.

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