Shape Optimization in 2–Dimensional Magnetostatics Using a Fixed Grid Approach and Algebraic Multigrid

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ABSTRACT

We deal with a fast and robust solution technique to optimal shape design. The governing physical problem is 2–dimensional magnetostatics, which leads to a boundary value Poisson problem discretized using linear nodal finite elements. The computational domain is fixed and we optimize the shape of the interface between the ferromagnetic and air subdomains.

A common solution technique, cf. [2], preserves a discretization of the design interface so that shape perturbations are mapped to the remaining grid nodes. However, this moving grid approach significantly restricts the shape changes on fine grids, since some elements may tend to flip after the deformation. Typically, all the shape perturbations happen at a coarse discretization level and at finer levels the initial shapes, which are prolongations of the optimized shapes from previous levels, hardly improve. Note that a geometric multigrid turned out to be a proper preconditioner for the arising linear systems.

To overcame the drawback, we propose an alternative approach, where we freeze the discretization grid throughout the optimization at each level and the design shapes are allowed to intersect the grid elements. The shape changes are mapped to the material coefficients. A clear advantage is an unlimited design space at all levels. The price for that is a lower approximation order of the finite element solution, a local nondifferentiability of the shape–to–coefficients map and a slightly more expensive assembling of the stiffness matrix while looking for elements intersected with the design interface. Here an algebraic multigrid preconditioner, cf. [1], performs very efficiently, which is no longer the case of the geometric multigrid. Regarding the overall computational complexity in the optimization, in numerical comparisons our fixed grid approach turns out to be superior to the moving grid approach.

References:

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