Homogeneous Finite Discrete Time Markov Chain

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Definition 1. *Let elements of $T \in \Re^{n \times n}$ be non negative and $Te = e$, where $e = (1, ..., 1)^T \in \Re^n$. Then we call T the stochastic matrix.*

The basic motivation for the study of homogeneous reducible Discrete Time Markov Chain (DTMC) is a quantitative risk and reliability analysis for Railways signaling systems see [5, 3, 4] . A natural property of the risk model is a presence of several independent sets of persistent - absorbing states. The sets represent the fundamental classes of the system hazards. The probability characteristic of the transitions to these classes is the issue of the risk analysis. This is the reason why it is necessary for us to study the reducible homogeneous finite DTMC.

Definition 2. *A finite Markov chain is stochastic process, which moves through finite number of states, and for which the probability of entering a certain state depends only on the last state occupied.*

Stochastic matrices are used for a description of Markov chains

Suppose that $\{X_m \mid m = 0, 1, ...\}$ is a finite homogeneous Markov chain on the states S_1, ..., S_n. Let $T \in \Re^{n \times n}$ be its corresponding transition matrix. More information on stochastic processes and Markov chains can be found in [1, 8]. From our point of view we are interested in the following characteristics : long time behavior, probabilities of reaching some states from other states, and, if such transitions are possible, the time which is necessary to reach these transitions.

We show possibilities for computing such characteristic as long time probability of process to be in state i, overall probability of reaching state i from the state j and mean times and variations of mean times to reach state i from the state j. We both concentrate on existence and computability of desired values. Special interest is devoted to possibility to compute desired values for both reducible in irreducible transition matrices. Some examples considering reducible cases are shown.

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