Convergence issues of some classes of IAD methods in computing Markov chains

Ivana Pultarová

Czech Technical University in Prague, Faculty of Civil Engineering; Thákurova 7, 166 29 Prague, Czech Republic; ivana@mat.fsv.cvut.cz

Abstract

The iterative aggregation - disaggregation (IAD) methods present a multilevel approach in solution a stationary probability distribution vector of a finite discrete time Markov chain represented by an irreducible stochastic matrix B. They consist in combining some basic iteration with solution a certain coarse scale problem.

The present work introduces some new results on the convergence properties of some classes of the IAD methods. We study the methods where the basic iteration corresponds to a polynomial matrix iteration, where the polynomial p fulfils $p(1) = 1$. It is shown that when $p(t) = \alpha t + (1 - \alpha)$, $\alpha \in (0, 1)$, then the algorithms converges in local sense for any irreducible matrix B. The existence and uniqueness of the fixed point is proved for $\alpha \in (0, 1\rangle$. In the case $p(t) = t$, the local convergence is obtained for any irreducible matrix B with a positive diagonal. In addition to it, when B contains at least one positive row, then the asymptotic convergence factor can be estimated.

New examples of divergence of the IAD methods are introduced for $p(t) = t$ or t^2. Also some observations concerning rapid convergence, i.e. obtaining the exact solution within finite number of steps, are presented.

The introduced theory is based on sparsity structures of stochastic matrices contrary to the quantitative analysis of convergence properties of the IAD methods for nearly completely reducible Markov chains. In order to find the boundary line between the local convergence and divergence in local sense, some new questions and hypothesis are formulated.

Keywords

Iterative aggregation - disaggregation methods, Markov chains, Stationary probability distribution vector.

References:

Marek, I. and Pultarová, I. A note on local and global convergence analysis of iterative aggregation-disaggregation methods. To appear in Linear Algebra Appl.

Stewart, W. J. (1994). *Introduction to the numerical solutions of Markov chains.* Princeton University Press.