Transfer of the boundary conditions for boundary value problems for selfadjoint differential equations of $2n$th order.

Jiří Taufer,

Department of Applied Mathematics,
Faculty of Transportation Sciences, CTU Prague
Prague 1, Na Florenci 25, Czech Republic

e-mail: `taufer@fd.cvut.cz`

The idea of the transfer of conditions for a very general boundary value problem for a system of differential equations with inner and transition conditions originates in various methods. The present paper is devoted to a special problem and the methods are chosen in order to exploit the additional information, above all the symmetry and some sign properties.

Consider the boundary value problem for a selfadjoint differential equation of $2n$th order

$$\sum_{i=0}^{n}(-1)^i(p_{n-i}(t)y^{(i)}(t))^{(i)} = q(t) \qquad \text{a.e. in} \quad (a,b), \tag{1}$$

where the coefficients of the equation satisfy the following requirements:
$q(t), 1/p_0(t) \in \mathcal{L}\ (a,b)$, and $p_i(t) \in \mathcal{L}\ (a,b)$ for $i = 1, ..., n$.

The quasiderivatives are defined to the following manner:
$y^{[k]}(t) = y^{(k)}(t) \qquad \text{for} \quad k = 0, 1, ..., n-1,$
$y^{[n]}(t) = p_0(t)y^{(n)}(t),$
$y^{[n+j]}(t) = p_j(t)y^{(n-j)}(t) - (y^{[n+j-1]}(t))', \quad \text{for} \quad j = 1, ..., n.$
Put $x_i(t) = y^{[i-1]}(t)$ for $i = 1, ..., 2n$ and the vector $x(t) = (x_1(t), ..., x_{2n}(t))^T$.

Consider the boundary condition for the differential equation (1) in the form $W_1x(a)+W_2x(b) = w$, where W_1 and W_2 are square matrices of order $2n$ and the vector w has $2n$ components. The matrices W_1 and W_2 have to satisfy certain conditions in order that the boundary problem may be selfadjoint and positive semidefinite. Under these assumptions we will describe the algorithm based on the method of the special transfer of boundary coupled conditions.