ON IDEAL AND WORST-CASE GMRES

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Abstract

The convergence analysis of the GMRES method for solving linear algebraic systems $Ax = b$ is still an open area of research. If A is normal, then GMRES solves a certain approximation problem on the spectrum of A. For nonnormal matrices, the situation is much less clear. A possible approach for investigating the GMRES convergence behavior in the nonnormal case is to concentrate on the *ideal* GMRES approximation $\varphi_k(A)$ that represents an upper bound on the *worst-case* GMRES residual norm $\psi_k(A)$,

$$\psi_k(A) \equiv \max_{\|v\|=1} \min_{p \in \pi_k} \|p(A)v\| \leq \min_{p \in \pi_k} \|p(A)\| \equiv \varphi_k(A).$$

Some nonnormal matrices A are known for which $\psi_k(A) < \varphi_k(A)$ for certain k. However, it is still unknown whether $\psi_k(A) = \varphi_k(A)$, or at least $\psi_k(A) \approx \varphi_k(A)$ for larger classes of nonnormal matrices.

In this talk we study the relation between ideal and worst-case GMRES. We summarize known results and prove that worst-case GMRES starting vectors satisfy so called "cross equality". This phenomenon has been first observed by Zavorin in his Ph.D. thesis. We also present our latest results in investigation of ideal and worst-case GMRES for a Jordan block.