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Solving Sequences of Linear Systems by Preconditioned Iterative Methods

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In this talk we will address the problem of constructing preconditioners for solving sequences of systems of linear algebraic equations. Such sequences arise in many applications like computational fluid dynamics and structural mechanics, numerical optimization as well as in solving non-PDE problems. For example, discretized nonlinear equations $F(x) = 0$ by the Newton or Broyden-type method for $F : \mathbb{R}^n \to \mathbb{R}^n$ lead to a sequence of problems

$$J(x_k)(x_{k+1} - x_k) = F(x_k), \tag{1}$$

for $k = 1, \ldots$ where $J(x_k)$ is the Jacobian evaluated in a current iteration x_k or its approximation [7], [8]. The solution of such systems is the main bottleneck in many applications mentioned above. Apart from improving solvers of individual systems there is a strong need for reduction of costs of more subsequent linear systems by inexact solvers and by sharing some of the computational effort. In the following we will consider solving the linear systems by a preconditioned iterative method.

One way to reduce the overall cost for solving all systems is to skip some Jacobian evaluations as in the Shamanskii combination of the Newton's method and the Newton-chord method [4], [13]. In this way we get more systems with identical matrices. If an iterative method is used, decrease of work for Jacobian and function evaluations and possible reuse of preconditioners is typically balanced with increase of work spent in more iterations. Another important special case is solving systems with multiple right-hand sides, see, e.g., [11], [16], [6], [14]. Approaches that attempt to recycle Krylov subspaces which are only approximately equal were studied in [12] and [9]. A further possibility to improve a sequence of runs of a preconditioned iterative method for a sequence of systems is to update previous preconditioners by Broyden updates.

The purpose of this talk is to introduce a new approach for updating direct [15] or inverse [1] preconditioners before the iterative method is started. Consider two subsequent linear systems and let us call quantities related to the first and second system old and new quantities, respectively. We are interested in efficient computation of the new preconditioner. In a matrix-free environment the system matrices are available only implicitly. Then we may use the nonzero structure of the old preconditioner and a few matrix-vector products with vectors determined by this structure to get the new preconditioner [5]. If both the old and new matrix are available explicitly (computed, e.g., by finite differencing) we generalize diagonal updates from [2] and [3], see also [10], to rank-k updates of a factorized preconditioner.

For direct methods there are many techniques for updating sparse factors which were mainly developed for the simplex method of linear programming. Our case is different since we are interested only in cheap approximate updates.

The final goal of this ongoing work is to capture influence of large entries in the difference between the old and new matrix by simple updates of the old preconditioner. The new technique will be compared with alternative approaches to improve solving sequences of linear systems.

This is joint work with Jurjen Duintjer Tebbens (Institute of Computer Science, Academy of Sciences of the Czech Republic).

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