Augmented systems in the potential fluid flow problem

Miroslav Tůma

Institute of Computer Science, Academy of Sciences of the Czech Republic, Pod vodárenskou věží 2, CZ-18207 Prague 8, Czech Republic, e-mail: tuma@cs.cas.cz

Mixed-hybrid finite element discretization of the Darcy's law and the continuity equation which describe the potential flow problem in porous media leads to a saddle-point problem with the symmetric and indefinite system matrix of the following block structure [1].

$$\begin{pmatrix} A & B & C \\ B^T & & \\ C^T & & \end{pmatrix}.$$

Several approaches for solving these problems have been considered recently. They range from a pure iterative solution based on the preconditioned conjugate gradient or MINRES methods, block elimination based on the Schur complement to the dual variable approach and consecutive iterative solution of the resulting system projected on the null space of the matrix block $(B, C)^T$. (see, e.g., [2], [3], [4] and [5]). In this contribution, our work on both theory and algorithms in this field will be presented. In addition, we will describe the real-world application which provides these saddle-point problems. The results were obtained in joint work with Mario Arioli, Miroslav Rozložník and Jiří Maryška.

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