

Chapter six

Open problems

Repeatedly appearing motives:

- Nonlinear problems in numerical linear algebra;
- Backward stability;
- Intermediate quantities and accuracy of final results;
- Theoretical results link computational cost and numerical stability;

- Analysis of iterative methods: Convergence bounds, rounding error estimates are tools, not the goals;
- CG, Lanczos – Gauss Q of R-S integral; Scaled TLS fundamentals - GMRES: Unexpected, revealing links, analytic and computational (finite precision arithmetic) parts deeply connected.
- Lanczos, MGS GMRES: Loss of orthogonality means convergence.

Possible work (in progress):

- RS integral model of FP CG and Lanczos;
- Instability of Krylov subspaces;
- Core problem in FP arithmetic, its use for regularization;
- Regularization property of Ksp methods;
- Using GMRES parametrization in [Arioli, Pták, S - 98] for mapping the sets of problems;
- Restarted CGS GMRES ...

The goal is **understanding**, and putting it into service of practical computations.

From practical point of view, we need

- Analytical theory of acceleration (preconditioning),
- Reliable and efficient stopping criteria,
- Theoretical justification of numerical stability.