On Sensitivity of Jacobi Matrix to Small Perturbations of the Spectral Data

Zdeněk Strakoš

Institute of Computer Science, Academy of Sciences of the Czech Republic,
Pod Vodárenskou věží 2, 182 07 Praha 8, Czech Republic,
and
Faculty of Mathematics and Physics, Charles University, Prague
email: strakos@cs.cas.cz.

The contribution is based on a joint work with
Dianne O'Leary, University of Maryland,
and Petr Tichý, Institute of Computer Science AS CR, Prague and TU Berlin

Abstract

We consider a problem of sensitivity and numerically stable computation of the entries of the Jacobi matrix from its eigenvalues and the first elements of its normalized eigenvectors. This represents the classical inverse problem in the discrete Gauss-Christoffel quadrature calculation [1, 2] and its perturbation part is equivalent to the problem of sensitivity of the Lanczos recurrences to the spectrum and the components of the starting vector in the individual invariant subspaces [8]. With the connection to orthogonal polynomials and the Gauss-Christoffel quadrature, the problem has a long and very interesting history, see [7]. It has been investigated in many independent, and sometimes rather isolated, lines of development. This can be illustrated on the fact, that the fundamental contribution by Dirk Laurie [4, 5], motivated by the earlier work by Gragg and Harrod [3], has practically been unnoticed in the numerical linear algebra literature.

This contribution briefly reviews different lines of development and puts the problem into a broader context of the sensitivity of the Gauss-Christoffel quadrature [7] and numerical stability of the conjugate gradient method [6].

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