Balancing Incomplete Factorizations for Preconditioning (Do we know standard matrix decompositions?)

Miroslav Tůma

Institute of Computer Science

Academy of Sciences of the Czech Republic

joint work with

Rafael Bru, José Marín, José Mas

Universidad Politécnica de Valencia.

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Outline

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Preconditioned iterative methods

Solving large, sparse SPD systems by iterative methods

$$Ax = b$$

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- As usual, should be cheap, fast to compute, implying fast converging preconditioned iterative method
- but also: sufficiently robust
- sparse enough
- providing just sufficient approximation of the algebraic problem, and not more if this makes computations faster.

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2. Drawing attention to some approaches which exploit info on matrix inverse.
3. Presenting an approach based on a new way to decompose the input matrix and not on preprocessings, postprocessings, additional frameworks or modifications

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Incomplete decompositions

Trivial paterns

Incompleteness based on pattern or on values?

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A) Very simple patterns for cheap / cache-efficient preconditioners?

Incomplete decompositions

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Incompleteness based on pattern or on values?

A) Very simple patterns for cheap / cache-efficient preconditioners?

Example: banded pattern: BCSSTK38, $n = 8032$, $nz = 181,746$

bandwidth (full)	iterations
1	426
3	821
5	648
9	1638
15	792
1011	105
1311	56
1511	†
3111	35
4111	18

Incomplete decompositions

Matrix-based patterns

B) Matrix-based patterns for preconditioners?

Incomplete decompositions

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Example: pattern of $A = \text{pattern } (L + L^T), \;\; L = tril(A)$

Well known: error R of the decomposition $A = LL^T - R$ satisfies:

$$r_{ij} = 0 \;\; \text{if} \;\; (i,j) \in \text{pattern}$$

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Well known: error R of the decomposition $A = LL^T - R$ satisfies:

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As above, error outside the prescribed pattern can be arbitrary, if (linear system, PDE, etc.) model allows this.

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Example: Pattern of powers of A

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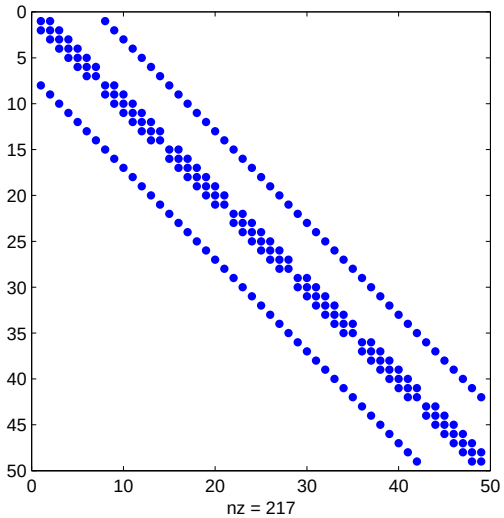

ILU(0)

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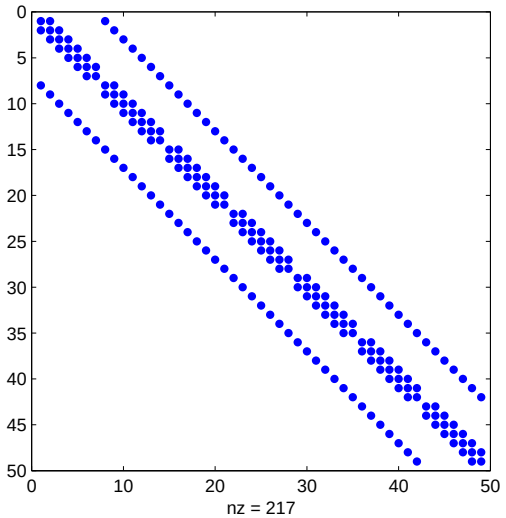

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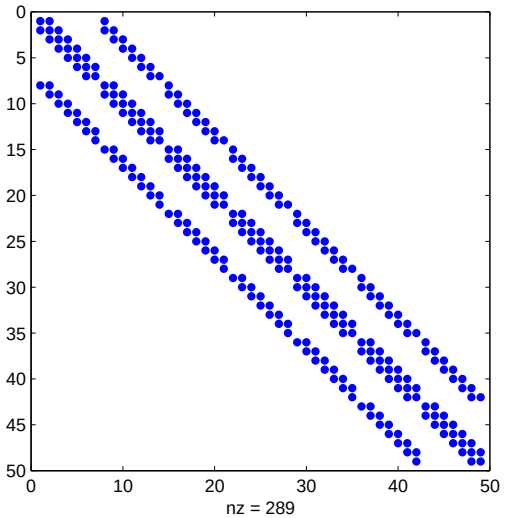

ILU(1)

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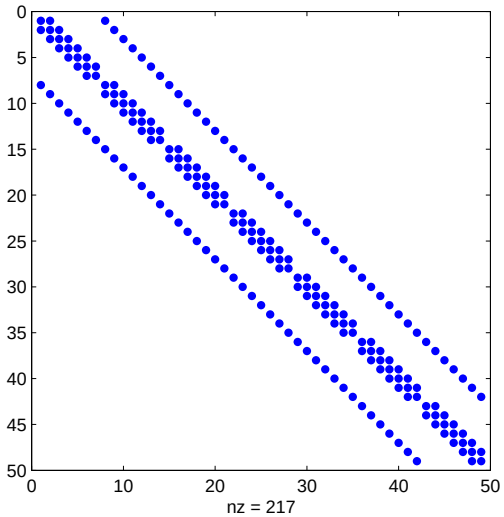

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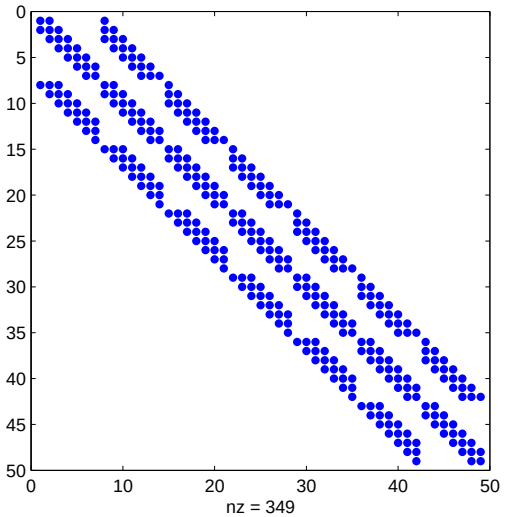

ILU(2)

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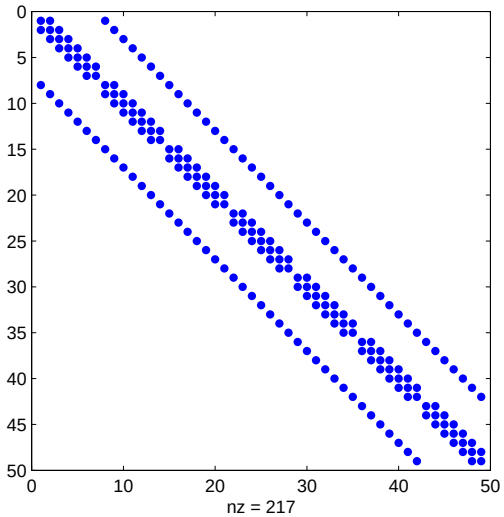

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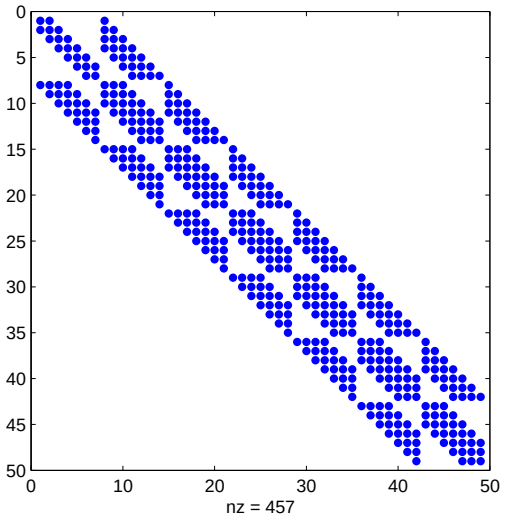

ILU(3)

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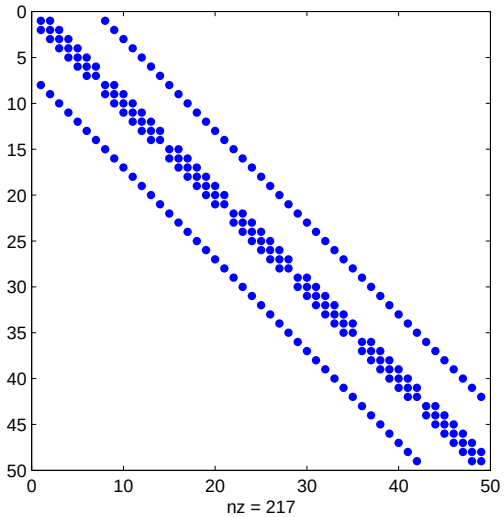

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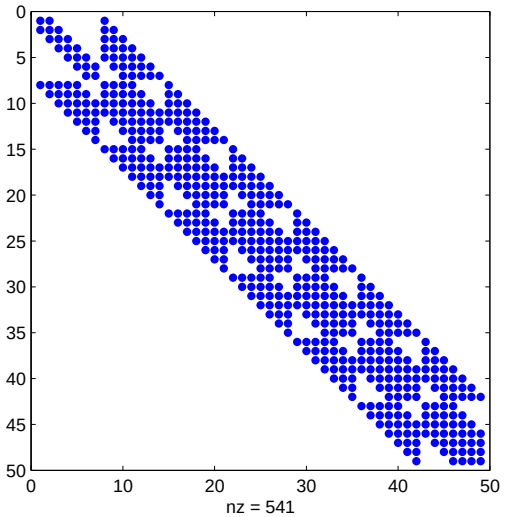

ILU(4)

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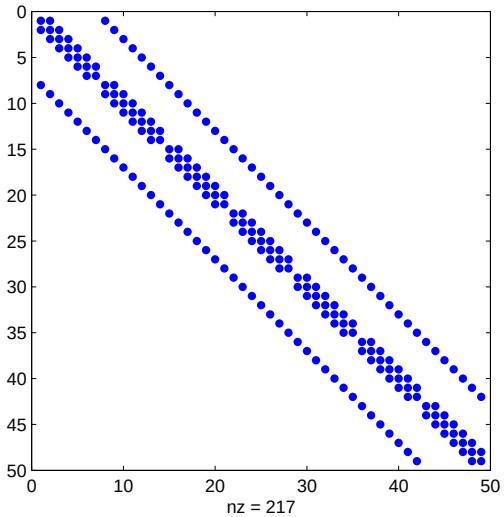

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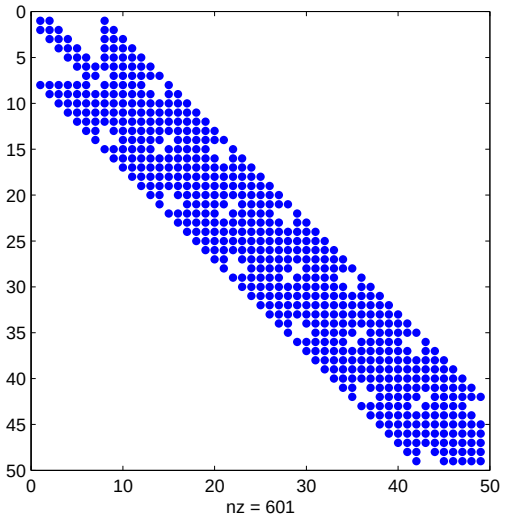

ILU(5)

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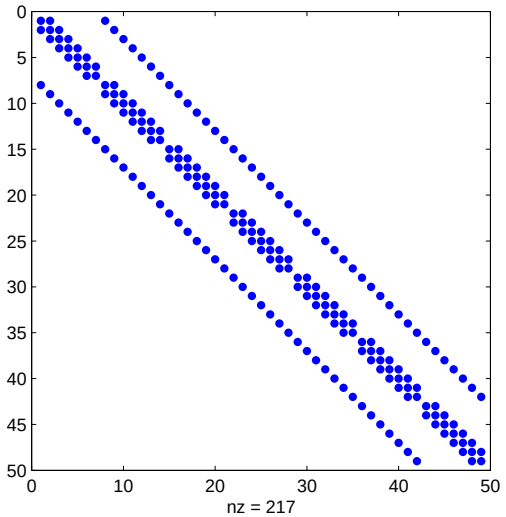

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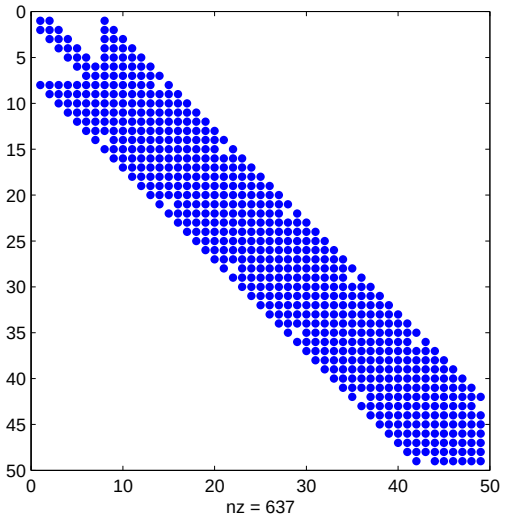

ILU(6)

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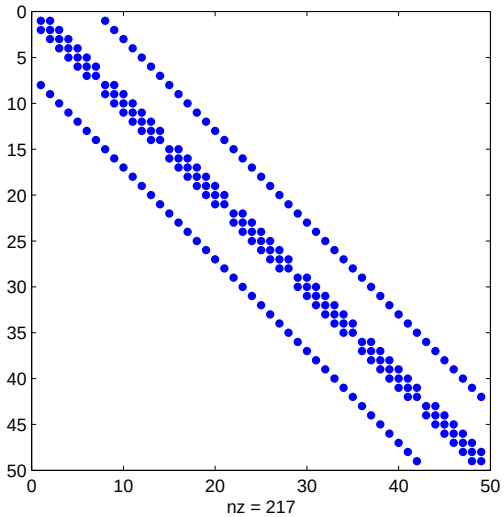

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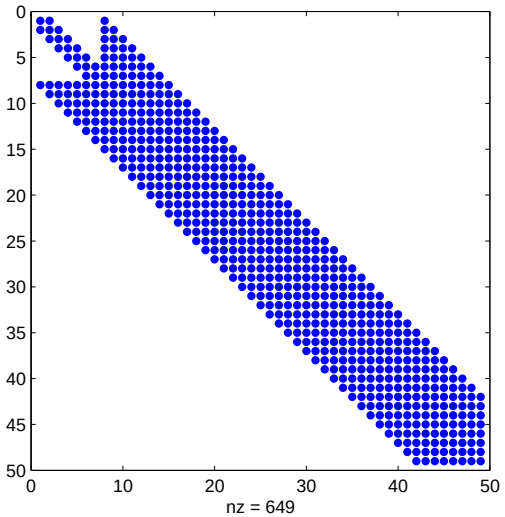

ILU(7)

Incomplete decompositions

Enhanced matrix-based patterns: using levels

- Fast computation (Hysom, Pothen, 2001)
- Typically expensive to apply for modest number of levels (powers of A)

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Example: Matrix ENGINE, $n = 143,571$, $nz = 2,424,822$

levels	size prec	iterations
0	2,424,822	523
1	4,458,588	300
2	7,595,466	199
3	12,128,289	115
4	18,078,603	87
5	25,474,380	54
6	34,153,746	45
7	43,861,328	46
8	54,276,063	36

Incomplete decompositions

External enhancements

D) Preprocessing, postprocessing, modifications

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0	2,424,822	523	2,424,822	439
1	4,458,588	300	4,394,040	214
2	7,595,466	199	6,509,826	159
3	12,128,289	115	8,859,522	96
4	18,078,603	87	11,292,927	66
5	25,474,380	54	13,664,157	49
6	34,153,746	45	15,891,321	34
7	43,861,328	46	†	†
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Similarly: postprocessings, diagonal/offdiagonal modifications based on sizes of entries

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precond / precond. size	its
Jacobi	810
IC(0)	> 1000
23,838,704	> 1000
30,047,027	> 1000
37,809,756	> 1000

Incomplete decompositions

Values (continued)

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- Next: IF via/with inverses. But, see also work of Saad and Bollhöfer, 2002.

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IF via approximate inverses

RIF motivation

RIF (Robust incomplete factorization)

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  Factor L of $A = LDL^T$ is $L = AL^{-T}D^{-1}$
- It can be easily retrieved from this inverse factorization

$$\Downarrow$$

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From L^{-1} we can get L (from $\widehat{L^{-1}}$ get $\hat{L}$)

IF via approximate inverses

RIF implementation

Note: $l_{kj} = \dfrac{\langle e_k, A\ell_j \rangle}{d_k} \equiv \dfrac{\langle \ell_k, A\ell_j \rangle}{d_k}$ for $k \geq j$

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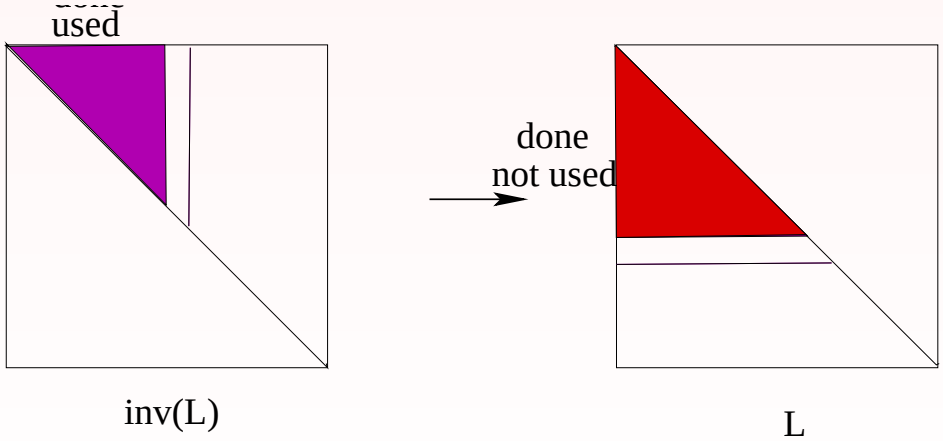

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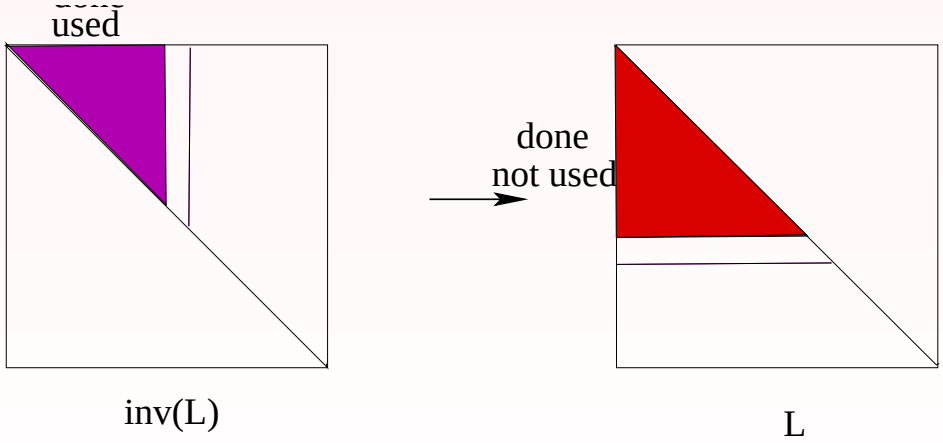

One way tranfer of information

Outline

IF with approximate inverses

$(I - A^{-1})^{-1}$ biconjugation

- Consider

$$A = I + \sum_{k=1}^{n} e_k (a_k - e_k)^T$$

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IF with approximate inverses

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- Apply n Sherman-Morrison updates to get A^{-1}.
  (Bru, Cerdán, Marín, Mas, 2003)
- The process for $R = (r_k)$, $V = (v_k)$, $D = \mathrm{diag}(d_1, \ldots, d_n)$ for $k = 1, 2, \ldots, n$:

$$r_k = e_k - \sum_{i=1}^{k-1} \frac{v_i^T e_k}{s r_i} r_i \quad , \; v_k = (a_k - e_k)_k - \sum_{i=1}^{k-1} \frac{(a_k - e_k)_k^T r_i}{s r_i} v_i,$$

$$d_k = 1 + (a_k - e_k)_k^T r_k = 1 + v_k^T e_k.$$

IF with approximate inverses

$(I - A^{-1})^{-1}$ biconjugation

- Consider

$$A = I + \sum_{k=1}^{n} e_k (a_k - e_k)^T$$

- Apply n Sherman-Morrison updates to get A^{-1}.
  (Bru, Cerdán, Marín, Mas, 2003)
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- $I - A^{-1} = R D^{-1} V^T$, R unit upper triangular.

IF with approximate inverses

balancing L and L^{-1}

Theorem

(Bru, Mas, Marín, T. 2007) For an SPD A, let there exist the decomposition from above

$$A^{-1} = I - RDV^T \tag{1}$$

and let $A = L\bar{D}L^T$ be the LDL^T decomposition of A. Then

$$V = L\bar{D} - L^{-T},\ R = L^{-1},\ \bar{D} = D.$$

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IF with approximate inverses

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- They can influence each other even in the exact case, purely by the decomposition (Bru, Mas, Marín, T. 2008).

IF with approximate inverses

BIF experiments

Example: matrix PWTK, n=217,918, nnz=5,926,171

IF with approximate inverses

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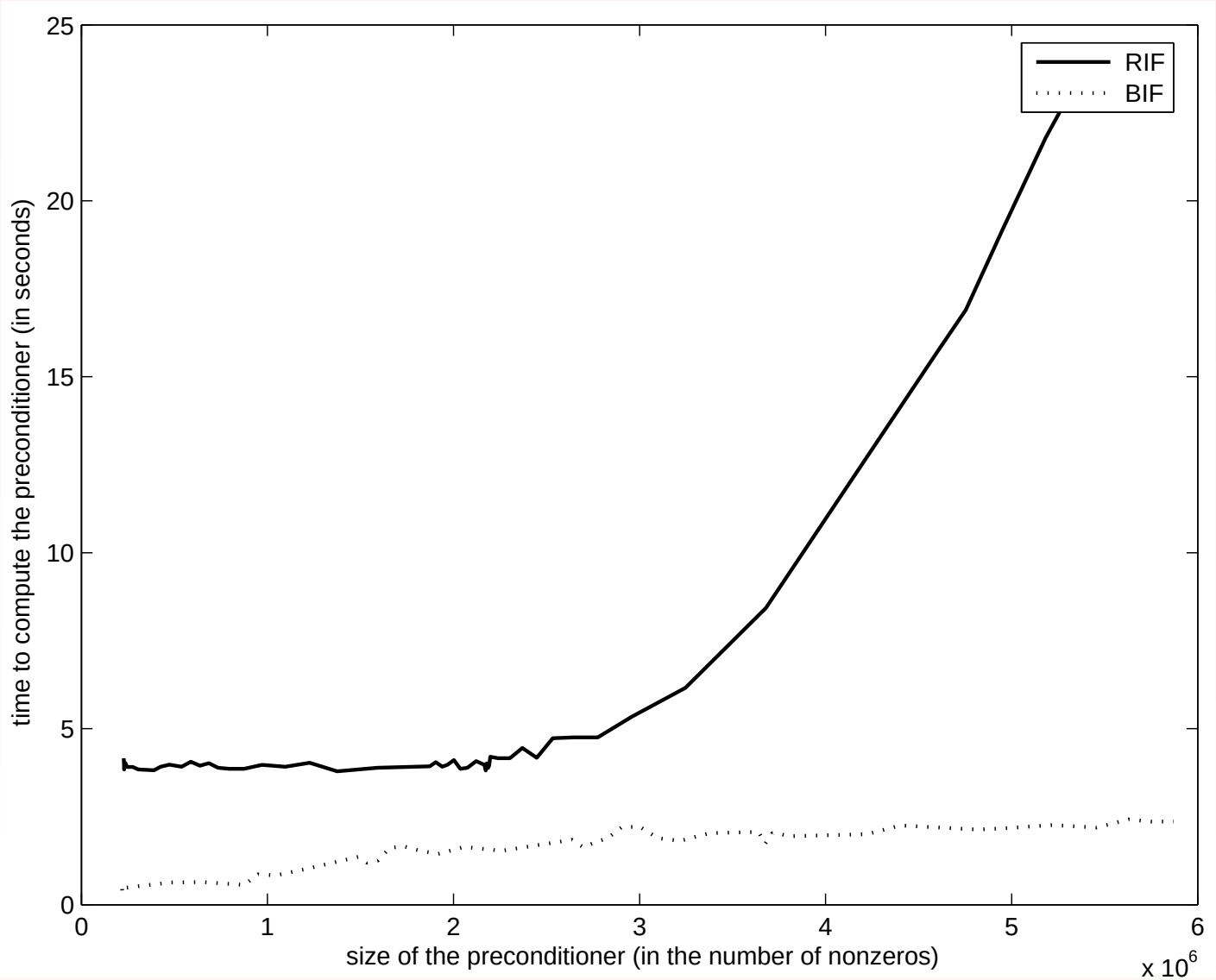

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- Convergence curve is later often flat if we run many iterations. Is the accuracy sufficient for solving sequences from nonlinear solvers?

IF with approximate inverses

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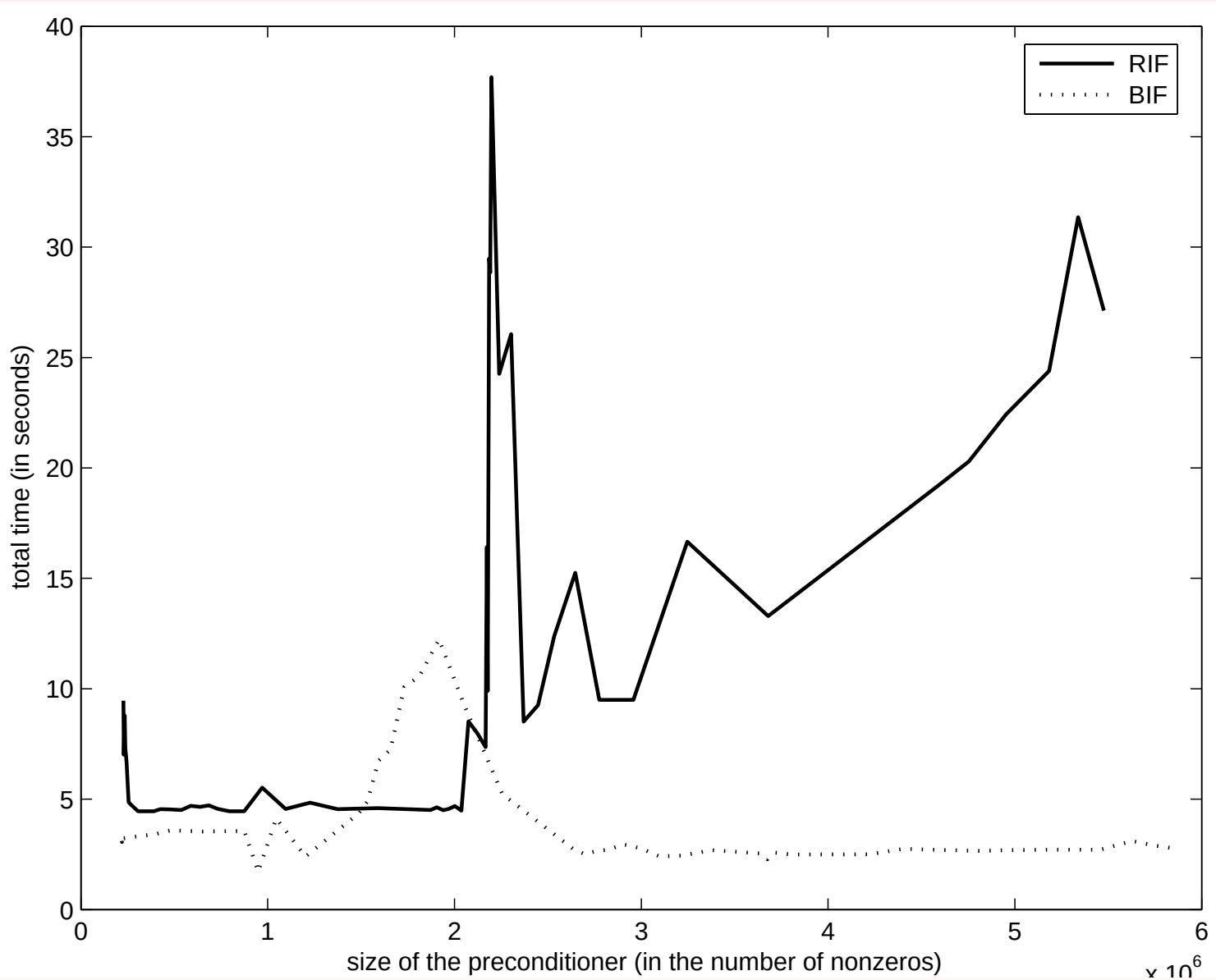

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